Vol. XX. No. 6 Whole No. 170

JUNE, 1920

SCHOOL SCIENCE AND MATHEMATICS

A Journal for All Science and Mathematics Teachers

Founded by C. E. Linebarger

SMITH & TURTON, Publishers

Publication Office, Mount Morris, Illinois

CHICAGO OFFICE, 2059 East 72nd Place, CHICAGO, ILL.

Published Monthly October to June, Inclusive, at Mount Morris, Illinois
Price, $2.50 Per Year; 30 Cents Per Copy

Entered as second-class matter March 1, 1913, at the Post Office at Mount Morris, Illinois, under the Act of March 3, 1879

SCHOOL SCIENCE AND MATHEMATICS

VOL. XX, No. 6 JUNE, 1920 WHOLE No. 170

RESEARCH IN PHYSICS.

Conducted by Homer L. Dodge.

University of Oklahoma, Representing the American Physical Society.

It is the object of this department to present to teachers of physics the results of recent research. In so far as is possible, the articles and items will be non-technical, and it is hoped that they will furnish material which will be of value in the classroom. Suggestions and contributions should be sent to Homer L. Dodge, Department of Physics, University of Oklahoma, Norman, Oklahoma.

EINSTEIN'S THEORY OF GRAVITATION FROM THE STANDPOINT OF THE TEACHER OF PHYSICS.

BY JOSEPH S. AMES,

President American Physical Society, Johns Hopkins University, Baltimore, Maryland.

Few theories in physics have aroused the interest created by Einstein's theory of gravitation. Yet it should be noted at the beginning that Einstein has not proposed a theory, in the sense in which that word is used ordinarily. He has, on the contrary, proposed a new method of approach to the subject of physics as a whole. He has succeeded, starting from certain postulates, in deducing formulae which are most general in their application and from which far reaching conclusions may be drawn. Among the consequences of his postulates are: 1. A system of giving numerical values to the intervals of space and time; 2. The concept of energy having mass, both in the sense of inertia and in that of gravity; 3. The conservation of mass (and therefore of energy); 4. The conservation of momentum; 5. A law of gravitation, which degenerates to Newton's law if the velocities of the particles of matter are small compared with that of light; and many others. The laws of electro-magnetism are shown to be in accord with the postulates.

The essential things to emphasize, then, are Einstein's postulates and his method. It cannot be stated too strongly that no *theory* is involved. The teacher of physics is not called upon to revise his concepts, his methods, or his theories. He will learn from Einstein's work that there are new correlations and that care must be taken in announcing certain experimental results to call attention to the conditions under which they are true. Further, and most important, of all, he will learn that in speaking of a length, an interval of time, mass, etc., he is always limited to the *measured* values of these; no abstract ideas of "absolute" values are involved. For instance, if one says that the length of a rod does not change when it is held in different positions, east and west, north and south, vertical, etc., what is meant is that, if the length is measured in these positions, the same numerical value is obtained; no meaning being possible in the query: "Has the length really changed?" Or again, if one says, that the mass of a body depends upon its velocity, what is meant is that the numerical value for the mass obtained by measurement varies with its velocity.

With this introduction, Einstein's postulates may be stated and discussed briefly. The first of these is: "Every law of nature which holds good with respect to a coordinate system K must also hold good for any other system K′, provided that K and K′ are in uniform movement of translation." This is sometimes called the principle of relativity. One has an illustration of it, if he is sitting in a railway train in uniform motion and another train passes; it is impossible for the observer to decide by looking at the other train whether it has the relative motion observed, or whether his own train has it, or whether it is divided between these two. What is observed is simply the *relative* motion. The second postulate is: "The velocity of light in a vacuum is a constant, regardless of the source of the light, e. g., a star, the sun, a light-house."

It may be proved that these two postulates lead to a method of defining our methods of giving numerical values to intervals of space and time. Thus, if the length of a rod is to be measured, the process is for the observer to select a standard of length, e. g., a meter-bar, place it alongside the bar, note the readings on the bar of the positions of the ends of the rod, subtract one reading from the other; the observer, the rod, and the bar having no relative motion. It follows that, if this process is repeated on different days, or in different places, or by different observers,

the numerical value obtained is the same, provided only the condition specified in the first postulate is satisfied. This method leads at once to the idea of two equal lengths; and two equal intervals of time are then, in accordance with the second postulate, such intervals as are required for light to traverse equal lengths. In other words the principle of superposition is adopted in the method of measuring lengths; and what is meant by the words "uniform velocity" is "velocity of light"—this is a definition, and this concept combined with that of equal lengths leads to the definition of equal intervals of time.

There are many other consequences of these first two postulates, and among them, two are perhaps the most important; one refers to mass, the other to energy. If we analyse our method of giving a number to the mass of a body, we see, without too much difficulty, that the number assigned by any observer is obtained by the solution of two sets of equations, one stating the Conservation of Mass, the other, the Conservation of Momentum, in which last the "velocity" is that measured by the observer. When the two postulates are applied to these equations, it is found that the number to be assigned to the mass of any particle increases as its velocity is increased. The exact formula connecting mass and velocity can be deduced; and this has been verified experimentally. A consideration of this formula also shows that it may be interpreted as proving that energy has mass; the relation being that, if E is the amount of energy in a certain space and if C is the velocity of light, the mass associated with that space is E/c^2. Expressed otherwise, as Einstein himself states it, this leads to the conclusion that "inert mass is nothing else than latent energy." This refers of course to mass as an expression derived from inertia, and does not carry with it any conclusion concerning weight. (Einstein proved later, as a consequence of his other postulates, that energy had mass also from the standpoint of weight.)

The first postulate is limited to uniform velocities, and, as Einstein says: "Must the independence of physical laws with regard to a system of coordinates be limited to systems of coordinates in uniform movement of translation with regard to one another? What has nature to do with the coordinate system that we propose, and with their motions?" Obviously, as he says "the choice of systems of coordinates for our descriptions of nature should not be limited in any way so far as their state of motion is concerned." This is his third postulate. We may

express it differently by saying that, if we wish to express a law of nature by a mathematical formula, this last must retain the same form regardless of the coordinate system used for the purposes of description.

This new line of thought led Einstein to consider that phenomenon of nature which we call gravitation. Its simplest illustration is the case of a falling body. We know that this force due to gravity is independent of the nature of the matter constituting the body; it might be called a "geometrical" property. There is another illustration of such a geometrical property in the case of centrifugal force. Consider a so-called "conical" pendulum, that is, a heavy bob suspended by a string and so set in motion that the bob describes a horizontal circle, and the string, therefore, a circular cone with a vertical axis. To an observer on the ground there are two "forces" acting on the bob; its weight and the tension of the string. Now, if an observer were to be attached to the rotating system so that he turns with it, being stationed at the axis, he would find it necessary in describing what he observed to introduce into his equations a term called the "centrifugal force." No one attaches any "reality" to this concept, it is purely a fictitious quantity, used to simplify our mathematics. Now this centrifugal force is entirely independent of the kind of matter; and Einstein raised in his own mind the question, if centrifugal force is a fictitious quantity, introduced into the equations and experienced by an observer moving with a certain system of coordinate axes, but not needed and not experienced by an observer on the earth, i. e., using a different system of coordinates, why is not the same also true of the force of gravity? In other words, may not a set of coordinate axes be so chosen that with reference to them an observer would be unconscious of gravity and therefore would not introduce this force into his equations of motion? It is obvious that, if an observer were in a large box falling freely towards the earth, he would not know that he was falling, he would not be conscious of any pull on his body. Einstein expressed this in his fourth postulate: It is possible to replace the effect of a gravitational field at any point at any instant by a mathematical transformation of axes. This he called the "Principle of Equivalence."

From these last two postulates come many important consequences. One of these is that the space around any material body is non-Euclidean; e. g., if one could measure with sufficient

accuracy the length of the diameter of a circle and its circumference, the ratio of these two lengths would not be exactly π (3.14159+). Einstein deduced formulae which describe the motion of a particle in a gravitational field due to another particle, formulae which reduce to Newton's formula for the same problem provided the velocity of the particle is small, but which give a statement in accord with observation for cases to which Newton's formula has not been found satisfactory, e. g., the motion of Mercury. Einstein also deduced a formula for the curvature of a ray of light—the nature of which as an electromagnetic disturbance is known—when it passes through the gravitational field due to such a body as the sun; this formula has been verified by the observations made at the solar eclipse of last year. Other consequences of the postulates of Einstein are the conservation of matter, the conservation of momentum and the assertion that energy is indistinguishable from mass as regards both inertia and gravitation.

The method used by Einstein was simply to apply mathematical processes, making use of his postulates. Thus by means of his first two, he deduced the relationship between the coordinates (x, y, z, t) of a point as observed by a man moving with the system K and those of the same point (x′, y′, z′, t′) as observed simultaneously by a man moving with the system K′. These transformations give at once a means of discussing velocities, masses, etc. When he came to his study of the problem of gravitation, he was led to investigate the forms of equations which do not change their character when there is a transformation of axes of coordinates, and, from among these forms, to select the ones which would satisfy our knowledge of gravitation. It is difficult to say whether one admires more Einstein's concept of the problem and the proper method of approach, or his skill in applying his mathematical knowledge to accomplish his purposes.

THE SCOPE AND OUTLOOK OF VISUAL EDUCATION.

By J. Paul Goode,
University of Chicago.

We have grown so accustomed to the printed page as the foundation of school education, so satisfied with the old routine of assigning so much text, and demanding a reaction from the pupil in some oral or written test, that it may be actually something of a shock to have a change suggested. Yet we discover that the printed page is one of the slowest means of presenting a wide range of information.

One of the oldest studies in the school, geography, was the first to take advantage of visual methods. The map is a system of shorthand in the presentation, to the eye, of space relations. And yet it has never been made to give its best service to the pupil. In all geography rooms, globes and maps are essential, but the very great value of the desk outline map, to be filled in by the pupil, in exercises and tests on distribution, is a largely untilled field in education. We are not only eye-minded, we are hand- or motor-minded; and working on a map has possibilities in education, largely overlooked.

Very early the geographer introduced the picture, as an aid in the presentation of his subject. But it is only in recent decades that the value of the picture has been demonstrated in many other lines as well as geography. A reading book in the lower grades nowadays is unthinkable without generous illustration. All the sciences and arts use the picture and the diagram, in increasing measure. The growing generosity of illustration by the current magazines and certain daily papers has been a godsend to the schools, wherever live teachers have undertaken to collect and use these pictures. One of the best services has been that of the National Geographic Magazine. Its collection of pictures, now over fifty thousand, are being reprinted, and made available at cost for individual pupil's use.

The success of this picture phase of visual education has been marked. But the pictures are as a rule too small for class use. This early led to the use of the projection lantern. But the lantern of early days was a cumbersome thing. The coming of electricity gave much more freedom, but even here the danger of open circuits, and the attention to the open arc have kept the equipment out of common use.

The coming of the Mazda filament lamp, however, has thrown

all barriers down. The lantern is coupled into any lamp socket, it can be safely managed by any child, the light is so intense that the darkening of the room is not a serious matter. By means of the reflectoscope, book and magazine illustrations become available also.

The lantern makes possible and profitable the use of many maps and graphs as well as pictures. A map can be copied into a lantern slide, and colored for a dollar or so, and thrown on the screen on a scale much larger than any printed map obtainable. This gives unlimited freedom to the instructor, for many maps which we may never hope to have published in large form could be used with profit in the classroom.

The graph is a device in visual education which has large possibilities, and is but little developed. A whole page of statistics can be thrown into the form of a curve, as for example, the production of wheat year by year for a generation, and the trend of production can be read at a glance. One may notice the conspicuous success of the Babson curves of business expansion and depression, and the growing use of graphics in many lines of business, to realize something of the possibilities of this form of visual education.

The finest service yet rendered in the schoolroom has been done by the stereograph. The photograph presents but two dimensions. But the stereocamera and the stereoscope work a miracle. They supply the actuality of binocular vision, and the third dimension is presented to the eye in vivid reality. The person who looks through the stereoscope looks upon the real mountain, looks into the depths of the real canyon, looks upon the actual statue, the actual cathedral.

The genius who learned how it could be made best to serve the purpose of schoolroom work is Mr. B. L. Singley of Meadville, Pennsylvania. In this development I have a personal, almost a fatherly, interest. It is now sixteen years since Mr. Singley came to me, to tell me of the opportunity for service which he could see in the stereoscope, and for consultation on the problem of adapting it to school needs. The needs of the classroom, and the possibilities of development of the stereoscope appealed to me, and my counsel and suggestions for development were gladly given. And so I have had always a personal pride in the development of the system. Mr. Singley discovered that the stereograph must be worked, but not overworked. It must help get the day's lesson, not get in the way

of the lesson. It must occupy the student without the attention of the teacher. It must lead the pupil to apply himself, and learn for the pleasure of learning.

The method is simple. An ample supply of stereographs is provided. The number in one standard set runs to 600. The subjects are chosen to cover the whole earth, and with selections so made as to cover many topics which will be studied in geography, in history, in literature. These stereographs are classified into all the topics where their use may be profitable; cross referenced, and indexed, and the whole study published in book form, as a Teachers' Guide, so that the teacher may find any stereograph available for teaching any subject, as easily as she can find a word in the dictionary, and can put her hand right on the required stereograph without a moment's delay.

Each stereograph has on the reverse side a description running to 250 words, written in an interesting style, and carrying the necessary information to the student. In use, the teacher puts out the stereoscope and one or two stereographs relating to the next day's lesson. Some time during the study periods of the day, each pupil will study the stereograph, read the description, and be ready next day to tell what he saw. It becomes a game to see who can stand and report in good English what he saw, looking through the window of the stereoscope into the reality beyond. At the end of the week, or when the review on the country or topic comes, the same views, in lantern slide form, are put before the whole class, and some pupil is chosen to stand before the class and discuss what one view presents, and other pupils report on other slides.

A real interest is aroused. Better teaching results. Live material is in hand always, for drill in geography, history, English. The success of the method is unquestioned. The sets of views are in use in thousands of schools all over the country. It is the best contribution yet made in visual education in America.

The stereograph arrives at perfection, in presenting the perception of solidity, and distance, the *third dimension* of the view. There is nothing to compare with it in this service, but it is a static world. Motion is absent. Yet *motion* is another "dimension," and the presentation of motion in the picture is an arrival at another apex of perfection. The gracefully moving animal, the rushing waves, the swaying trees, are all there, to the last perfect detail of motion.

Since education comes through arousing the interest of the child, and since the power of the movie to arouse interest is patent to all, it has occurred to many people to draft the movie into the service of the schoolroom. Every trial has shown some measure of success, but always some critical drawback has arisen to block progress. The flickering light on the screen is hard on the eyes. The projection machine is very expensive. It uses a large current, which may be dangerous, especially as it is likely to set fire to the film. That puts it under ban by the insurance interests, and an expensive housing or shelter is required. This restricts its use to the auditorium, and this in turn takes it out of the reach of class work. The films are very expensive, and for the most part have been made with the one aim of entertainment or of advertisement, so may not be satisfactory or even usable for purposes of instruction. In short, the whole matter up to the present moment seems like an exhibition of misfit effort, showing a lack of intelligent cooperation on the part of the interests directly involved.

And now comes another genius, in the person of Professor Forest Ray Moulton of the University of Chicago, one of the leading mathematicians of the country. Professor Moulton found that the standard projection machine is very inefficient, the screen being lighted only one-tenth to one-third of the time, the other nine-tenths or two-thirds of the time being in darkness. Hence the flicker. He invented a new movement, reversing the ratio of service, having the screen illuminated nine-tenths of the time, and the *flicker disappears.* Not only this, but the motions are smooth and natural, not jerky.

Since only about a tenth of the current hitherto used is required, the machine for a classroom can be coupled into any lamp socket and operated by a child. The machine can be stopped at will and held at any point for study and discussion with no danger of setting fire to the film.

With these advantages in hand a Society for Visual Education has been formed for the purpose of solving the problems in the adaptation of the cinema to purposes of instruction in the schools. All sorts of tests and measurements will be made, to find out the place and best service of each of the devices in visual education, in the administration of the school program.

Now let us make no mistake as to the efficiency of any or all the devices which may be used in visual education. No one of them or all of them will ever take the place of the live, earnest

competent teacher. Moreover the best of teachers will have to be initiated into the best methods of using the graphic material. All of the visual devices together will not remove the need of effort or work on the part of the pupil. The pupil's real achievement will be measured by the attention and effort of the pupil. But the visual helps will create interest, stimulate attention, and reduce effort so more ground may be covered in a given time. So also may a higher record of achievement be won by a larger number of pupils.

And this brings us to the economic phase of our quest. It will pay school boards to invest in the proven methods of visual instruction. The Racine, Wisconsin, schools in 1910 compared well with the schools of other cities of similar size, the country over. Their record of pupils failed at the end of the year was low—only one in ten of the pupils below the high school. A ten per cent failure was to be expected. In 1910 these schools began to adopt the stereographic equipment called the 600 Set, and the failures began to decrease. Rooms began to make a record of no failures at all during the year. In 1914 the Russell Sage Foundation made a wide study of "failures and promotions" and the Racine schools were recorded as showing an average of five per cent of failure. The survey also brought out the fact that in the schools where the new system of visual education was not used, the record still stood at ten per cent. In the 5,000 children in the Racine schools, between the kindergarten and the high school, cutting the failures from ten per cent to five per cent gave promotions to 250 pupils, who without the improved instruction would have been ranked as failures and would have been required to repeat the course. To have had 250 pupils repeating the course would have called for six or eight extra teachers, and extra rooms. On the basis of the average cost of a year's schooling, this promotion of 250 pupils was a saving to taxpayers of Racine of between $10,000 and $15,000 in the year. Think of what the saving to the whole country will be when visual education has been fully worked out and entered in all the schools.

There are in the common schools of the country at this time, in the grades below the high schools, over 20,000,000 pupils enrolled. The record shows over 15,000,000 in attendance. The average annual cost per pupil in these schools in 1914 was not far from $30 each. This cost has doubtless doubled since then. An average of ten per cent failure in this number gives

us about 1,500,000 pupils to repeat the work. This, at $60 per pupil, makes the very respectable sum of $90,000,000. Suppose now, the introduction of visual education could cut this failure record down to five per cent on the average. That would make a saving of $45,000,000, a prize well worth working for. Not only can this improvement be made in the grades, it can be made in some measure in the secondary schools as well. The equipment thus made ready may serve in Americanization work, in churches, and in Community Centers. This is a wide and magnificent opportunity for service. It is worthy the best brains and most serious effort of all of us.

THE AURORA OF MARCH 22, 1920.

By Lewis J. Boss.

In the early evening of March 22 there took place the most spectacular display of aurora borealis that has occurred so far this year. When first seen at about 7:30 p. m., a wide ribbon-like, waving curtain of green light extended in a huge arc across the northern sky, well over the constellation Cassiopeia. From this arch pulsating streamers shot towards the zenith and seemed to converge in a point directly overhead. At times the edges of some of the streamers and of the curtain bands would show a decided crimson tint, which would waver and flare like the flame of a dying fire. Maximum activity seemed to be reached at about 8:00 p. m. and then the display began to die down. Soon, however, the height and brilliancy of the streamers began to increase once more and by 8:05 p. m. it became evident that another outburst was about to take place, which would equal if not surpass the first.

The phenomena this time took the form of a wide wavy curtain of light, ever shifting and weaving in and out, over which rosy-pink patches of light traveled slowly toward the east and faded out, only to begin again at the north and repeat the performance. At 8:10 p. m. the magnetic needle showed marked perturbation, swinging toward the east five or ten degrees, and once showing a shift of twenty degrees. During some of the more brilliant flashes a distinct snapping or crackling sound could be heard. These manifestations continued for nearly ten minutes, after which their intensity rapidly diminished, and by 8:20 p. m. only a few pale green wisps of light remained quivering in the north. The aurora did not cease, however, but with occasional fits and starts continued until the early morning of March 23. One of the features of this exhibit was the occurrence of several intensely bright green areas of light almost sixty degrees from the horizon, and of a brightness equal to that of the full moon.

On March 23 I observed the sun and noted a row of five large spots and several smaller ones. This is a larger number of spots than I have seen on the sun lately. This was the best auroral display we have had here in two or three years, in fact I do not recall having seen an aurora which was as bright as this one in a long time.—[*Popular Astronomy.*

MAPPING BY AERIAL PHOTOGRAPHY.[1]

By W. H. Spurgin,

Hyde Park High School, Chicago, Ill.

No one, who has a most casual interest in maps, can glance at the ground from an airplane in flight without the feeling that he is looking at a vast colored map, which, in its wealth of detail of land and water, of railroads, highways and paths, of forest and cultivated ground, of buildings and towns, in its absolute accuracy and up-to-dateness, should be the envy and despair of every professional map-maker. So striking is this resemblance that it could hardly have failed to occur to balloonists years ago that a map could be very easily made with a camera.

Much has been written concerning the importance of aerial photography in the war; and the limits of this paper do not permit of a full discussion of that topic, interesting though it might prove. Yet, since the science of aerial photographic mapping owes by far the greater part of its growth to the war, we must glance for a moment at some of the uses to which it was put.

The Germans seem to have started using hand-held cameras early in the war with, of course, no attempt at systematic mapping, but these were quite unsatisfactory, largely because the leather bellows caved in under the strong air currents, and soon a metal camera with fixed focus and focal plane shutter was devised and attached to the fuselage. Plates were carried in magazines, and exposures and changes of plates made by hand. Development was rapid to a highly perfected type, using roll films, kept flat by suction, and capable, when adjusted for altitude and speed and set in motion, of making automatically 100 successive exposures at any desired time interval, the power for operation being supplied by a small fan propeller working like a wind-mill.

Complete photographic maps of the zone of activity formed the basis of operations in each sector, and squadrons of reconnaissance planes were employed keeping them up to date. In time of great activity large sections would be re-photographed every day, while particular points might be photographed every two or three hours, and the series carefully studied for any changes. Not only were these photographs extensively used at

[1]Read before the Earth Science Section of the Central Association of Science and Mathematics Teachers, November 29, 1919.

headquarters, but copies were given to infantry officers when an attack was planned, and to the artillery for checking up the effect of their fire. Photographic expeditions were carried far back into the enemy territory for the purpose of mapping lines of communication, camps, ammunition and supply dumps, flying fields, and the like. The occasional value of a single photograph is seen when we are told that on one occasion twelve French planes, carrying twelve pilots and twelve observers were dispatched at 15-minute intervals in an attempt to obtain one picture. Perhaps we should notice, before leaving this topic, that accurate base maps were continually used as a basis for, and a check upon, the photographic maps, they not being of sufficient accuracy otherwise.

The range of a flyer's vision is perhaps not commonly realized. By applying the theorem of the secant and its external segment as related to the tangent we find that, at an elevation of one mile, the horizon, in comparatively flat country, is 89 miles distant, and that that distance increases as the square root of the altitude. Thus, on a perfectly clear day a flyer 6500 feet above Lansing, Michigan, could easily see portions of L. Michigan, L. Huron, and L. Erie; while if Rohlf, on his record altitude flight last September, of 34,610 feet, had been over Lake Huron he could actually have seen in one sweeping glance all of Lakes Huron and Erie, more than half of Lakes Michigan and Ontario, and a considerable portion of Lake Superior.

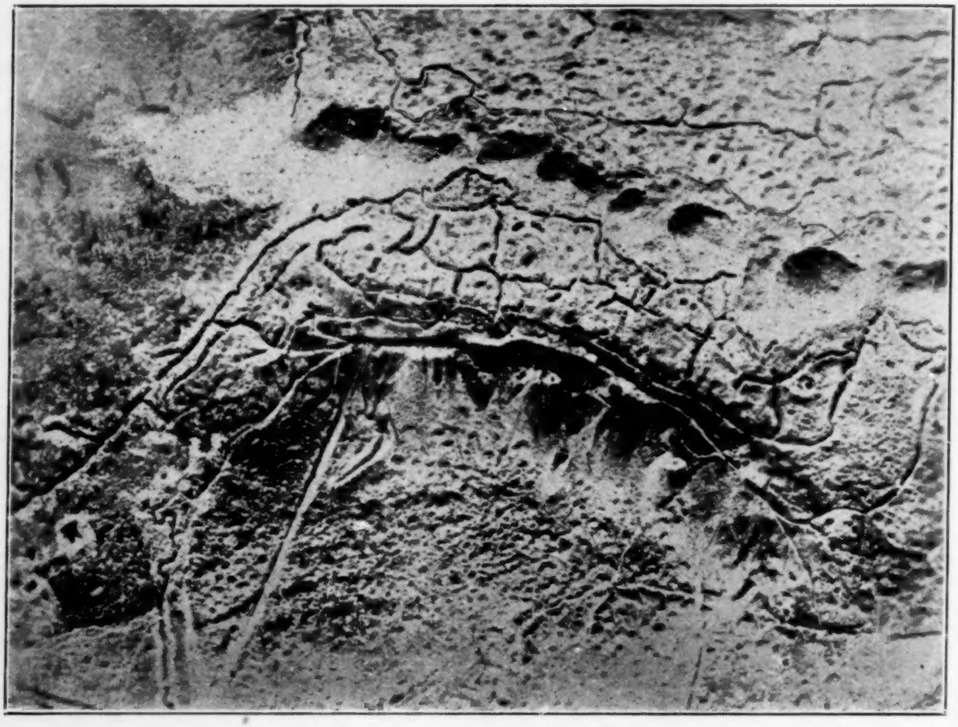
The area included within a single photograph depends, of course, upon the altitude, the focal length of the lens, and the area of the plate. While lenses of 48-in. focal length were used where a detailed view of some object such as an enemy battery was desired, and anti-aircraft guns made considerable altitude necessary, the most satisfactory lens for general mapping was the 10-inch. With this lens, and an elevation of 10,000 feet the image is approximately on the scale of 1-12,000. An 8x10-in. plate, then, would cover an area about 2 miles long and 1.6 miles wide. When a continuous map of a larger area is desired the pilot flies in a straight line, taking pains to retain the same altitude, and to keep the plane horizontal to avoid distortion, while the observer makes exposures at an interval calculated to give 1-4 to 1-3 overlap. The prints are then carefully matched and pasted together, forming a mosaic. As many strips as are necessary may be photographed in this way and a complete map built up of any desired area.

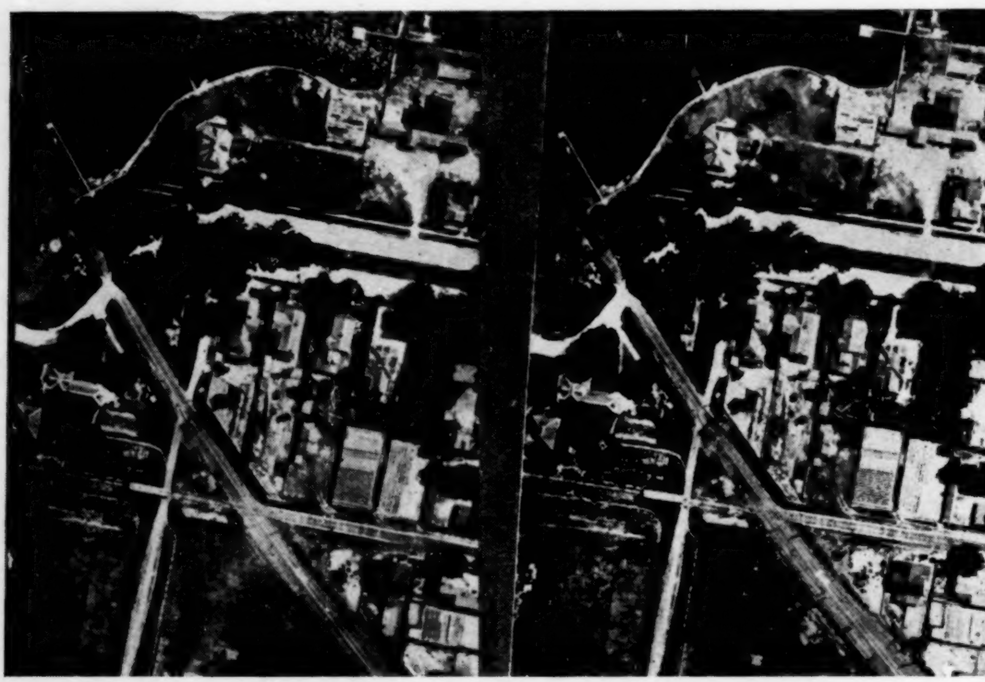
Very little aerial mapping has been done in this country.

During the war a few such maps were prepared by photographic units in training, the largest being of the vicinity of Ft. Sill, covering an area about 35 miles by 10 miles. Another, somewhat smaller, but of more interest to geographers, covered a portion of the alluvial fan region of Southern California. The work of mapping routes connecting the various flying fields was started but not completed. During the past summer a squadron was engaged in making a mosaic 20 miles wide of the route between Oklahoma City and Fort Sill, a distance of 80 miles. 30,000 plates will be exposed, and the completed map will be on the scale of about 1 foot to the mile. Another squadron is mapping coast-to-coast aerial routes.

It may seem at first thought that an aerial photograph, taken from vertically overhead might be held in any position for study. In this picture (see insert), the trench system runs along a pronounced crescent-shaped ridge; in front, a row of mine craters; behind, dirt thrown out from dugouts, and down the slope. If we turn the picture upside down I think you will observe a persistent attempt on the part of the trenches to disguise themselves as stone fences in a valley, while the mine craters and shell holes are neatly turned wrong side out. The explanation of the reversal of slopes seems to lie in the fact that in all of our experience with photographs of landscapes, as well as with landscapes themselves, the shadows fall, in general, down. If a picture is held so that the shadows fall up our mind refuses to accept the new situation and insists upon interpreting the slopes in such a way as to fit our pre-conceived notions of downward-falling shadows. The same effect may sometimes be seen on an ordinary terrestrial landscape view, particularly if it has well defined shadows and no prominent artificial objects.

Perhaps some of my hearers have been struck by the apparent flatness of the country in aerial photographs. It is true that all relief disappears at a comparatively low elevation unless the sun is at such a low angle as to bring it out by shadows. Much may be inferred from the course of roads and railroads, from the direction of plowing where contour plowing is followed, and from the streams, but nothing definite as to elevations. The writer spent the greater part of the summer of 1918 at Eberts Field in the rice district of Arkansas. The land is extremely level, but still the rice-growers find it necessary to throw up dikes along the contour of the surface to retain the standing water on the growing rice. When viewed from an elevation

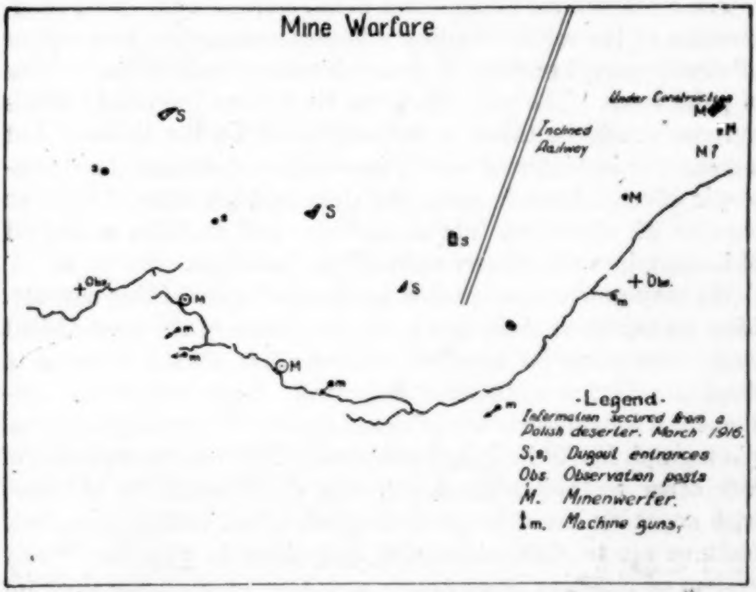




STEREOSCOPIC VIEW OF WATER FRONT AT HAMPTON, VA.

This picture may be cut out, mounted on cardboard, and viewed through an ordinary stereoscope.

of 2,000 or 3,000 feet as perfect a contour map as one could desire is formed by the dark green color of the rice as it grows on the dikes, contrasted with the brighter green of that growing between the dikes. If contours were everywhere so easily discerned the problem of showing relief in aerial photographs would be solved. But, unfortunately, resort must be had to other means.



We are all familiar with the principle of the stereoscope. Obviously an ordinary stereoscopic camera with lenses about 2 3-4 inches apart would be of no value in aerial work, for the effect practically disappears at a little more than 100 feet, and it would reveal no more relief than could be seen by the observer. But, if the lenses of such a camera are more widely separated, the stereoscopic effect is exaggerated. For our present purpose the separation would have to be greater than would be possible on an airplane; so the result is obtained by taking photographs in the same way as for a mosaic, except that more overlap is allowed. The portion that is common to both prints may then be mounted as a stereo. As a matter of fact, the stereo was of such value in military work that in much of the photographing an overlap of 1-2 was allowed so that the interpretation could be done largely by use of the stereoscope. Nor is it necessary to make careful mounts, as the prints may be run through a

cardboard with slits and quickly adjusted for any particular object. One or two examples will serve to illustrate the use of stereos in the war. Shell holes were frequently covered over with camouflage to conceal machine gun or battery positions. While these particular shell holes looked no different from others in an ordinary view, they immediately became conspicuous in a stereoscopic view by reason of their lack of depth.

The most striking thing about aerial stereos is the great exaggeration of the relief. Ordinary houses assume the proportions of skyscrapers, box-cars of grain elevators, and ordinary trees of palm trees. The rule was given that views intended for this purpose should be taken at an interval of 1-8 the altitude, but a greater or less interval merely increases or decreases the stereoscopic effect. Interchanging the right and left sides of a stereo changes all elevations into depressions and we have the effect of looking into the plaster casts of the buildings.

We may now turn our attention to the future of this new art. May we accept at their face value the claims of the most ardent supporters of aerial mapping to the effect that it is soon to displace all other methods of mapping? There can be no question that, for completeness of detail (apart from relief) the aerial photograph leaves little to be desired. Nor can we conceive of any other method whereby all this detail could be obtained with anything like the ease and speed. But, desirable as these features are in themselves, the first thing to consider in any system of mapping is accuracy, and here we open the door for a host of disturbing factors.

Let us look first at the physical difficulties involved in securing the picture. The camera is more or less rigidly attached to the fuselage which necessitates that the plane, at the moment of exposure, be perfectly horizontal; otherwise, the plate will not be parallel with the ground, and distortion will result. Furthermore, in making a mosaic, all the exposures must be made at exactly the same altitude. Both of these conditions call for very skillful piloting. To be sure, pictures taken at differing altitudes may be enlarged or reduced to the same scale, and even those taken at an angle may be rectified, but that requires an accurate base map of the region. Some device may be developed which will retain the camera in its proper position regardless of the attitude of the plane, but we must bear in mind the instability of the support itself. The use of small dirigibles instead of airplanes would to some extent obviate

this difficulty by reason of their ability to hover over a point, thus enabling the observer to work more leisurely.

Again, a low altitude requires either a wide angle lens to cover territory, or a much larger number of exposures. But high altitudes, above 10,000 feet for instance, involve temperature changes which are likely to disturb the focus and retard the action of the moving parts.

Fog and haze are a hindrance, but the use of color sensitized plates and color screens, which were developed by the Bureau of Standards during the war, have wonderfully reduced this difficulty.

The focal plane shutter introduces another error which may not be disregarded. This arises from the fact that the plate is not all exposed at once, which causes an elongation in the direction of flight.

Let us suppose, however, that by one device or another these technical difficulties have been surmounted. Certain inherent weaknesses still remain. In a hilly region the hilltops, being nearer the camera, will photograph larger. Thus, if the camera is 10,000 feet above a valley, and a neighboring hilltop is 2,000 feet above the same valley, the lengths of the images of equal distances on the two will be in the ratio of 8-10. Furthermore, hills not in the center of the picture will appear to lean away from the center, particularly so with wider angle lenses, and more noticeably so the farther away from the center.

In view of these shortcomings it seems quite certain that where accuracy, say of one part in 10,000, is a requirement, photographic mapping cannot supplant the more laborious methods. But that does not mean that it may not become a very valuable aid to these established methods. Given a base map with a sufficient number of control points, cultural detail may be gathered with amazing speed, completeness, and ease. Scarcely forty per cent of the area of the United States has been satisfactorily mapped. Surely much of the detail of the remaining work may be obtained from the air. Old maps may be brought and kept up to date. Changing coast lines may be mapped with sufficient accuracy and as frequently as necessary, or may be roughly surveyed to determine whether remapping is needed. It is even possible that the configuration of the bottoms of shallow bodies of water may be evidenced by varying shades, although the extent to which this will be possible is somewhat uncertain. Difficultly accessible regions, such

as swamps, may be mapped approximately. Preliminary surveys may be made of unmapped areas which will serve until accurate base maps can be prepared and the aerial map may then be transferred and corrected.

One of the oil companies which has large holdings in British Columbia had spent a considerable sum exploring a mountainous region in search of a pass through which a pipe line might be run. A year ago they were considering making an aerial survey. Whether the plan was carried out I am unable to state.

Considerable stand of wood-pulp timber was located during the past summer in and west of Labrador by an aerial reconnoissance party. Some 13,000 plates were exposed.

Those of you who have seen the stereos will realize that even topography may be sketched in roughly. It has been suggested that conspicuous monuments of known height might be erected over the region to be mapped, which would serve as standards. Instruments which have been in use in connection with stereoscopic mapping of some of our western mountains may then be used to determine the elevations of conspicuous points by measuring the stereoscopic effect, and the contours then sketched in.

Perhaps no more appropriate use could be found for the aerial camera than the mapping of air routes. One could ask for no better map of the country over which he is to fly than its picture. Nor would he be concerned with absolute accuracy. And there seems to be good reason for believing that air travel, on a commercial basis, will be largely confined to fairly definite lanes in order to take advantage of established landing fields. Here, certainly, photographic mapping has a field which is peculiarly its own, and a future which is limited only by the future development of aerial navigation.

THE GREAT POTASH DEPOSITS OF GERMANY.

The potash deposits of Germany, which were discovered by the Prussian Government in 1843, at Stassfurt, while boring for rock salt and which occur in upper layers of rock salt in the plains of northern Germany, have been estimated to occupy a volume of 10,790,000,000 cubic meters and to contain 20,000,000,000 metric tons of potash salts, corresponding to about 2,000,000,000 metric tons of potash (K_2O), a quantity sufficient to supply the world for 2,000 years at the present rate of consumption. These beds, according to the United States Geological Survey, Department of the Interior, were first exploited about 1860, and have furnished practically the entire world's supply of potash for many years.—[*U. S. Geological Survey.*

A NEW DIRECT READING VACUUM GAGE.

By W. H. Farr,

Central Scientific Company, Chicago, Ill.

When the Central Scientific Company began producing the Cenco-Nelson Vacuum Pump in quantities one problem which presented itself to the laboratory was that of a satisfactory routine test of the vacuum produced by this pump. The chief requirements of such a test are, that it must be quick, reliable, and "foolproof." The McLeod gage was used for some time, but there are some objections to its use as a routine test in quantity production. In the first place it is rather slow. It not only takes a little time to fill the bulb and empty it again but besides it is necessary to take a series of readings over a period of time to see whether or not the vacuum is improving. Furthermore the apparatus is rather delicate for routine use, and can be easily ruined by careless handling. If oil is accidentally allowed to get into the bulb, the cleaning and drying of the glassware involves considerable loss of time. In case the glassware is broken or if mercury is spilled, renewal or recalibration is rather expensive. In view of these facts, the laboratory began to investigate other means of measuring a vacuum, which would be more suitable for our conditions.

The variety of effects displayed by electrical discharges in rarified gases suggested the idea of using some of these phenomena to indicate the degree of vacuum produced by a pump. The problem was to determine which of the characteristics of the vacuum discharge could be measured quantitatively with sufficient accuracy to serve as a measure of the vacuum.

Of the two sizes of the Cenco-Nelson Pump (2 stage and 3 stage) the 2 stage is guaranteed to produce a vacuum of .10 mm. and the 3 stage of .05 mm. Occasionally a 3 stage pump will be found which will go as low as .02 mm.

Before a vacuum of .10 mm. has been reached the Faraday's dark space has become well defined, and occupies a considerable portion of the tube. The positive column is broken up into well defined striations, which become larger and fewer as the vacuum improves. It was first attempted to use the length and form of the positive column as the quantity to be measured. This gives a fair indication of the vacuum, but is not sufficiently reproducible to serve as a reliable test.

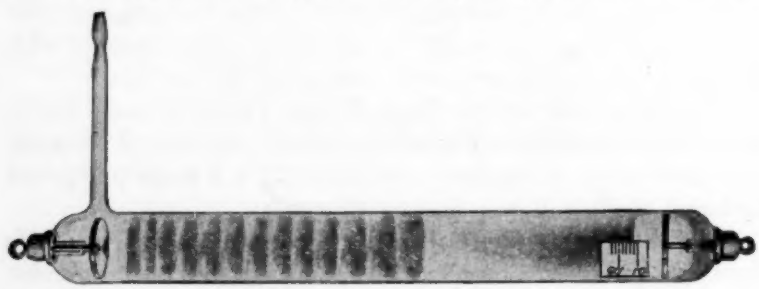
It is affected by such factors as the size and proportions of the tube, the form of the electrodes, the dryness of the tube and the

frequency of the current. This unreliability is especially noticeable in the neighborhood of .05 mm. and above, which is just where the greatest accuracy is desired.

Another phenomenon which was investigated was the appearance of the apple green fluoresence which indicates the beginning of the production of cathode rays. The difficulty here was that there is nothing about this color which can be measured quantitatively, and its appearance is affected by some of the conditions of the test, such as the condition of the tube, the shape of electrodes and the strength of the spark coil.

An attempt was made to test the vacuum in the tube by measuring the current flowing through the tube with a hot wire milliammeter, but this method was abandoned. It was also attempted to use a spark gap in parallel with the tube, the length of the gap to be adjusted so that it would spark over at the desired vacuum. The resistance of the tube proved to be too low for this method to be a success.

It was finally found that the most satisfactory characteristic of the vacuum discharge to be measured quantitatively is the Crooker dark space. This dark space begins to appear around the negative electrode at a vacuum of about one millimeter and increases up to a vacuum of about .02 or .01 when it becomes too hazy and indefinite to measure. The dark space is most easily measured in a tube having disk shaped electrodes. In such a tube it appears as a cylindrical shaped space with a well de-

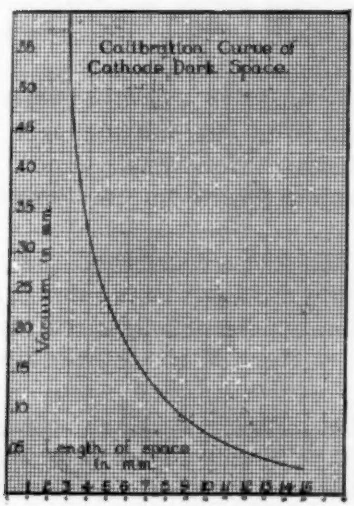


fined edge, as shown in Fig. 1, whose length can be measured with an accuracy of about 1-2 mm. It will be noted that the range over which this dark space is measurable covers the range of the vacuum produced by the Cenco-Nelson Pump.

To determine the reliability of the measurements of this dark space as an index to the degree of vacuum, it was necessary to determine how much it is affected by the conditions of the test,

such as the individual characteristics of the tube, voltage of the battery, frequency of the vibrator, voltage of the coil, etc. It was found that the length of the dark space is not affected by the diameter of the tube but there is a certain relation between the diameter of the electrodes and the diameter of the tube, and also between the length of the tube and its diameter, which gives the most clear and definite reading.

To determine if the dark space varies from one tube to another depending on the individual characteristics of the tube, a number of tubes were made up, as nearly indentical as possible in form and dimensions. These were calibrated by the McLeod gage and the measurements compared. It was found that in tubes of the same form and dimensions the length of the dark space is an accurately reproducible quantity. The effect of the voltage of the coil and the frequency of the vibrator were also investigated. The only effects of changes in the voltage of the coil is to vary the intensity of the discharge, it having no effect on the length of the cathode dark space. The same is true of the frequency of the vibrator.



The relation between the length of the dark space and the vacuum is shown by the curve in Fig. 2. It will be noted that the rate of change in length becomes more rapid as the vacuum becomes higher. This is a decided advantage, as it makes the accuracy of the readings increase as the vacuum increases. In the tests of the Cenco-Nelson Pumps the readings come near

the lower end of the curve, at which point it is easily possible to graduate the scale on the tube to read .01 mm. directly, which is a satisfactory degree of accuracy for this test.

During the development work on this test, all the work was done in the dark room. When it was perfected, however, a shade was constructed which made it possible to use the equipment in daylight. This was in the form of a wooden box whose dimensions were slightly larger than those of the tube, and which was painted black inside and outside. The tube is observed through a wide slot in the front side. The tube is held in spring clips, so that it is easily removable, and the connections are made through binding posts on the outside.

In conclusion, the specific advantages which are claimed for this test are the following:

1. As the connections are shorter and there is a smaller volume of air to exhaust, the desired vacuum is reached more quickly than when using a McLeod gage.

2. The reading is instantaneous, whereas some little time is required to read a McLeod gage.

3. This test gives a continuous indication of the vacuum, while with the McLeod gage it is necessary to take a series of readings to determine whether the vacuum is improving or not.

4. As the scale on the tube is direct reading, it requires no experience or practice to interpret the result, nor is there any source of error due to a zero setting.

5. The accuracy of this method increases with the degree of vacuum.

6. This test does not require the handling of mercury or expensive glassware, making it more suitable for a routine test.

7. The complete apparatus is light and portable.

8. In case of breakage of the glassware, renewal is not expensive.

9. A number of testing sets can be operated from the same coil and battery, it being only necessary to duplicate the vacuum tube.

10. The accuracy of the test being independent of the voltage or frequency, there are no meter readings or corrections.

CONSULTATIVE WORK IN GEOGRAPHY AS A MEANS OF IMPROVING ITS TEACHING.

By Alison E. Aitchison,
Cedar Falls, Ia.

It is being realized more and more fully that to improve, immediately, the work which is being done in geography in the elementary schools, some effort must be made to help the teachers while they are in service. Many of them have neither opportunity nor time to stop teaching and take courses in college or normal school. Many more have attended such schools and have taken no courses in geography, either because it was not required or because they considered it a subject which anyone could teach without special preparation. So into the grades they went from high school and college to teach geography, they themselves not having touched it, or anything akin to it since they left the seventh grade. In the meantime, great changes have taken place, not only in the subject matter of geography but in the methods of presenting it.

Endeavoring to meet these conditions, which exist everywhere, the Extension Division of the Iowa State Teachers College sends out through the state a member of the geography faculty, to do what is termed "Consultative Work in Geography." Of course the number of places which can be taken care of in any one year is very limited. A schedule is arranged allowing the consultative worker to spend from two to four days in a school system, the number of days varying with the size of the town.

On meeting the city superintendent, the first morning, a general program is laid out. This is often changed greatly as individual teachers ask for help along some special lines. The visit of the consultative worker is much more satisfactory to all, when the superintendent and teachers have discussed the plan beforehand and know the purpose of the consultative work and what they may hope to gain from it.

Generally the first day is spent in visiting classes. This gives the visitor an idea of the course of study and the quality of the work being done, besides enabling him to meet the individual teachers and judge to some extent their problems. The program is so arranged that classes in geography can be visited at every period from nine till four. Thus by the time school closes one has a fairly good idea of the work in two or three different buildings. The superintendent and the principal of

the building visited are generally present at all recitations.

After school, from three-thirty or four o'clock to five or five-thirty, a conference is usually held with the teachers, principals and superintendent. At this time some of the general principles of geography teaching are discussed, and the teachers bring up questions pertaining to their own particular grades, dealing either with subject matter or with methods of presentation.

The second day is most frequently spent in demonstration teaching, either before all the teachers of a certain grade at one hour and those of another grade at another hour or before all the teachers of one building at one time. In larger places two days are given to this work, since it is not easy for all the teachers to meet at one building. The demonstration teaching is followed always by a discussion, sometimes a very lively one.

Slightly different problems present themselves at each place. Some schools are working out a new course of study and the superintendent requests help on that. Effective work may be done in that way as few superintendents have either the time or the detailed knowledge of geography to enable them to plan a course in the subject which will be really helpful to inexperienced teachers. Most of the courses simply say "for the fall term course — pages — to — in — the first book of —."

In other schools little supplementary material and few wall maps are on hand, and suggestions for their selection can be made. Often conferences are held with individual teachers, who ask for help in the improvement of their own classroom work.

The experience of many Iowa schools shows that here is a method that may be applied to any state, yet so far but little advantage has been taken of it, of reaching the teacher just when she feels the greatest need of help and is willing to ask for it. If we believe that geography can bring something to the child, that no other subject can, we must make immediate efforts to see that it is made to occupy a more significant place in the curriculum of every elementary school.

RESEARCH IN CHEMISTRY.

Conducted by B. S. Hopkins,
University of Illinois, Urbana.

It will be the object of this department to present each month the very latest results of investigations in the pedagogy of chemistry, to bring to the teacher those new and progressive ideas which will enable him to keep abreast of the times. Suggestions and contributions should be sent to Dr. B. S. Hopkins, University of Illinois. Urbana, Ill.

THE POSITIVE ELECTRON AND THE BUILDING OF ATOMS.

By William D. Harkins,
The University of Chicago.

(*Continued from May*)

The composition of the heavy atoms may be found in a paper by the writer now in print in the *Physical Review*.

Why The Atomic Weights Are What They Are.

The writer has developed a theory in addition to that given above, which tells why the atomic weights vary on the whole as they do, but, to save space, the theory will be given for the atoms of even nuclear charge alone, that is for those whose nuclei are compounds of alpha particles. As shown above the alpha particle has a weight of 4.00 and a positive charge of 2 (net charge). From three to eight of these particles will unite to form a complex nucleus, and it is evident that the weight of the nucleus must be twice its charge, that is twice the atomic number. However 9 alpha particles do not unite (at least only in minute amounts) but ten alpha particles will unite if two negative cementing electrons are used to bind on one extra alpha particle. On account of the presence of these two negative electrons, the nuclear charge is only 18, or just what it would be if 9 alpha particles *alone* had formed it. The atomic weight is thus four plus twice the atomic number. This may be put in the form of an equation.

$W = 2(N+n)$, where W is the atomic weight, N is the atomic number, and n is the number of cementing electrons in the nucleus (n is zero for light atoms), that is they are similar to the electrons which escape as beta particles from the nuclei of radio-

active atoms. Why is it that when eight alpha particles can unite without any cementing electrons, it is necessary that such cementing electrons, which always add on in pairs shall be present in the nuclei of heavier atoms. This is because alpha particles repel each other, since they are positively charged, when at a distance, but when very close together attract on account of the action of their positive and negative charges as couples, though they still repel due to their positive net charge, but when only a few, up to 8, alpha particles are united the attraction exceeds the repulsion. However with more than 8 the repulsion becomes too great for the formation of a stable nucleus. The ratio of negative to positive electrons in the alpha particles is one to two, or ½. When the positive charge in the nucleus gets too large for the formation of a stable nucleus, the nucleus may be stabilized by the addition of the helio group described above, since the ratio of negative to positive electrons in the helio group is one to one, and its introduction increases the ratio of the negative to the positive electrons in the nucleus. Thus in element 16, sulphur, the ratio of negative to positive electrons is 0.5, as it is in practically all of the lighter atoms, but by the addition of one alpha group and two cementing electrons (a helio group), this ratio is raised to 0.55, which gives the requisite stability for the existence of the argon nucleus. Then alpha particles will add on, several in direct succession, until again the increase in the positive charge on the nucleus, makes it necessary that 2 more cementing electrons be added in one step. Thus the cementing electrons increase in steps, and at each step the atomic weight increases by eight instead of by 4 as in the simple addition of alpha groups. For this reason the atomic weight increases more rapidly among the heavy than among the light atoms.

Evolution of the Atoms.

The elements have thus been found to fall into two series: first, those of even, and second, those of odd, atomic number. Now, if the theory presented for the structure of the atoms is correct, then it should be possible to find some difference between the two series with reference to their properties. Since, however, this part of the theory refers specifically to the structure of the nuclei of the atoms, and not to the arrangement of

the external or non-nuclear electrons, it is evident that this difference should not be found in those properties due to the external electrons, that is in the chemical or physical properties. On the other hand, the difference should be found in any properties inherent in the nucleus, and the only property, aside from mass and weight (from which our system has been developed), which has thus far been discovered, and which is due to the structure of the nucleus of the atom, is that of atomic stability. Thus, if an atom loses outer electrons, it does not change its atomic number, and therefore does not change to another element, but if it loses nuclear electrons, it does change its nucleus, its atomic number is changed, and the atom is said to disintegrate—that is, it changes into the atom of another element.

Our theory therefore indicates a probable general difference in stability between the even- and odd-numbered elements. A consideration of the radioactive elements indicates that those which have odd atomic numbers have either shorter periods, or else are at present unknown. Now unfortunately there is no known method of testing the stability of the elements of low atomic number, but it might seem, at first thought, that the more stable atoms should be the more abundantly formed, and to a certain extent this is undoubtedly true. If then, at the stage of evolution represented by the solar system, or by the earth, it is found that the even-numbered elements are more abundant than the odd, as seems to be the case, then it might be assumed that the even-numbered elements are on the whole the more stable. However, there is at least one other factor than stability which must be considered in this connection. The formula of the even-numbered elements has been shown to be nHe'. Now, since that for the odd-numbered elements is $nHe'+H_3'$ it is evident that if the supply of the H_3' needed by the elements was relatively small at the time of their formation, not so much material would go into this system, and this would be true whether the H_3' represents three atoms of hydrogen or one atom of meta-hydrogen (ν).

In studying the relative abundance of the elements the ideal method would be to sample one or more solar systems at the desired stage of evolution, and to make a quantitative analysis for all of the ninety-two elements of the ordinary system. Since this is evidently impossible, even in the case of the earth, it might be considered that sufficiently good data could be obtained

from the earth's crust, or the lithosphere. However, the part of the crust to which we have access is relatively so thin, and has been subjected to such far-reaching magmatic differentiation, and to such extensive solubility effects, that it seems improbable that the surface of the earth at all truly represents its composition as a whole. The meteorites, on the other hand, show much less evidence of differentiative effects, and undoubtedly represent more truly the average composition of our planetary system. At least it might seem proper to assume that the meteorites would not exhibit any special fondness for the even-numbered elements in comparison with the odd, or, vice versa, any more than the earth or the sun as a whole, at least not unless there is an important difference between these two systems of elements, which is just what it is desired to prove. A study of

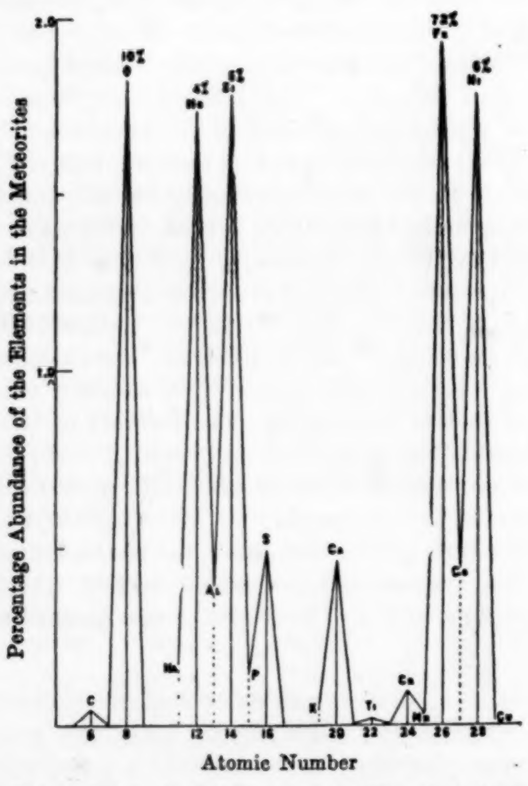


FIG. 1. The Periodic Variation in the Abundance of the Elements as the result of Atomic Evolution. The data are given for 350 stone and 10 iron meteorites, but the relations are true for meteorites in general. Note that *ten* elements of even atomic number makes up 97.59 per cent. of the meteorites, and *seven* odd-numbered elements, 2.41 per cent., or 100 per cent. in all. Elements of atomic number greater than 29 are present only in traces.

the compilations made by Farrington, by Merrill, and by other workers of analyses of meteorites, has given some very interesting results.

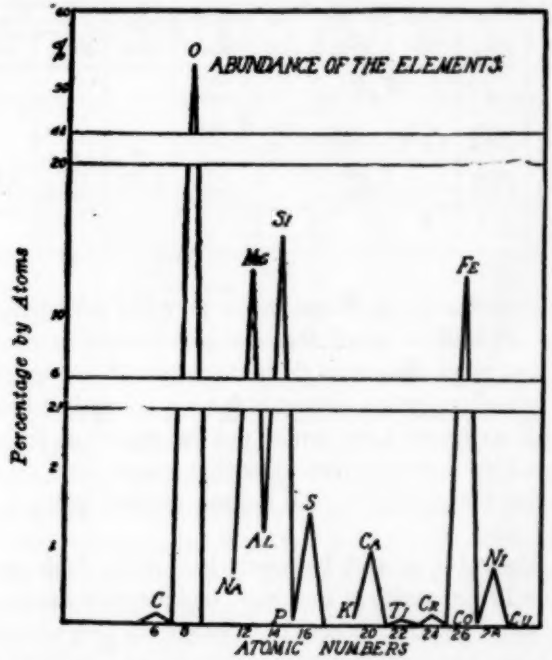


FIG. 2. The abundance of the elements in the meteorites. Every even-numbered elemen is more abundant than the two adjacent odd-numbered elements. (Percentage by weight Average for stone and iron meteorites.)

The results show that in either the stone or the iron meteorites the even-numbered elements are very much more abundant than the odd. Thus in the iron meteorites there are about 127 times more atoms of even atomic number than of odd, while in the stone meteorites the even-numbered elements are about 47 times more abundant. If we average the 125 stone and 318 iron meteorites given by Farrington, it is found that the weight percentage is 98.78 for the even- and 1.22 for the odd-numbered elements, or the even-numbered elements are about 81 times more abundant.

If we consider these same meteorites, 443 in all, and representing all of the different classes, it is found that the first seven elements in order of abundance are iron, oxygen, silicon, magnesium, calcium, nickel and sulphur, and *not only do all of these elements have even atomic numbers, but in addition they make up 98.6 per cent. of the material of the meteorites.*

Table IV (see Figs. 1 and 2 also) gives the average composition of these meteorites. The numbers before the symbols are

TABLE IV.

Average Composition of Meteorites Arranged According to the Periodic System.

Series	Group 1 Odd	Group 2 Even	Group 3 Odd	Group 4 Even	Group 5 Odd	Group 6 Even	Group 7 Odd	Group 8 Even	Group 8 Odd	Group 8 Even
2				6C 0.04%		8O 10.10%				
3	11Na 2.17%	12Mg 3.80%	13Al 0.39%	14Si 5.20%	15P 0.14%	16S 0.49%				
4	19K 0.04%	20Ca 0.46%		22Ti 0.01%		24Cr 0.09%	25Mn 0.03%	26Fe 72.06%	27Co 0.44%	28Ni 6.50%
	29Cu 0.01%									

the atomic numbers, and those below give the percentages of the elements. *It will be noted that the even-numbered elements are in every case more abundant than the adjacent odd-numbered elements.* The helium group elements form no chemical compounds, and are all gases, so they could not be expected to remain in large quantities in meteorites. For this reason, and also because the data are not available, the helium or zero group is omitted from the table.

From this table it will be seen that while *high percentages,* as great as 72 per cent. in one case, *are common among the even-numbered elements,* the highest percentage for any odd-numbered element is less than one per cent. (0.39 for aluminium).

If we now turn to the composition of the earth, it is found that the atoms of even atomic number are about ten times more abundant in the surface of the earth than those which are odd. Also, all of the six unknown elements except the element of atomic number 72, eka-caesium, eka-manganese 1, eka-manganese 2 (dwi-manganese), eka-iodine and eka-neodymium, have odd atomic numbers.

While the relative abundance of the elements in the lithosphere is undoubtedly much affected by differentiation, there is one group whose members are so closely similar in chemical and physical properties, that they would be much less affected in this way than any other elements. These are the rare earths. The only difficulty in this connection is that of making an accurate estimate of the relative abundance. In this the writer has been assisted by Professors C. James and C. W. Balke, but any errors in the estimate should not be attributed to them. In the table, which includes beside the rare earths a number of elements adjacent to them, the letter *c* indicates common in comparison

with the adjacent elements, and *r* represents rare. *ccc* represents a relatively very common element, etc. The comparison is only a very rough one, but it indicates that the even-numbered elements are in general more abundant than the odd-numbered ones which are adjacent.

TABLE V.

The Predominance of Even-numbered Elements Among the Rare Earths.

Atomic Number.	Abundance.	Element.	Atomic Number.	Abundance.	Element.
55	c	Caesium	63	rr	Europium
56	ccc	Barium	64	r	Gadolinium
57	c	Lanthanum	65	rrr	Terbium
58	cc	Cerium	66	r	Dysprosium
59	r	Praseodymium	67	rrr	Holmium
60	c	Neodymium	68	r	Erbium
61	rrr	Unknown	69	rr	Thulium
62	c	Samarium			

The above results may be summarized in the statement that *in the formation of the elements much more material has gone into the elements of even atomic number than into those which are odd,* either because the odd-numbered elements are the less stable, or because some constitutent essential to their formation was not sufficiently abundant, or as the result of both causes.

It is easy to see, too, that in the evolution of the elements, the elements of low atomic number and low atomic weight have been formed almost exclusively, and this indicates either that the lighter atoms are more stable than those which are heavier, or else that the lighter atoms were the first to get the material, and their stability was at least sufficient to hold it.

It is possible that the heavier atoms have been formed in larger amounts than now exist, and that their abundance has been reduced by atomic disintegration. It is of course evident that the radio-active elements are now disintegrating, but the radioactive series of elements includes only those of atomic number 81 (thallium) to 92 (uranium); and lead (82) is the end of the series as now recognized. For our purposes, however, we still call the atoms of atomic numbers 1 to 29 the lighter atoms, and from 30 to 92 the heavier atoms. The following table indicates that when defined in this way the lighter atoms are extremely more abundant. In the table the weight percentages are given, but it is evident that if these same figures were calculated to atomic percentages they would show even smaller values for the heavier elements. The table shows that although the heavy atoms have been so defined as to include more than twice

as many elements as the light atoms, their total abundance is so small as to be relatively insignificant. The data are taken from estimates by Clarke and by Farrington.

TABLE VI.

Illustrating the Large Proportion in Various Materials of the Elements of Low Atomic Numbers (1-29)

Material	Percentage of Elements with Atomic Numbers 1-29	30-92
Meteorites as a whole	99.99	0.01
Stone meteorites	99.98	0.02
Iron meteorites	100.00	0.0
Igneous rocks	99.85	0.15
Shale	99.95	0.05
Sandstone	99.95	0.05
Lithosphere	99.85	0.15

It is thus seen that *so far as the abundance of the elements is concerned, the system plays out at about element 30*, and it is of great interest to note that it is just at this point that other remarkable changes occur. For example, *up to this point nearly all of the atomic weights on the oxygen basis are very close to whole numbers.* On the other hand the elements with higher atomic numbers (28 to 92) *have atomic weights which are no closer to whole numbers than if they were wholly accidental. Also, just at this point the atomic weights cease to be those predicted by the helium-hydrogen theory* of structure presented in this paper (Table I.). This does not mean, however, that the helium-hydrogen system fails at this point, but that the deviations in the atomic weights for the elements of higher number are produced by some complicating factor. This is easily explained as due to the fact that these elements are practically all mixtures of isotopes. It is quite possible that radioactive disintegrations have proceeded downward in the system as far as iron, and that iron is the end of a disintegration series. If this were true, it would explain the great abundance of iron in the meteorites. In whatever way we may average the analyses of the materials found in meteorites or on earth, the two most striking elements from the standpoint of abundance are oxygen, the most abundant of the elements of very low atomic number (8), and iron, which has the highest atomic number (26) of any very abundant element.

The fact that the elements which have heavy atoms (atomic numbers 30 to 92, or more than two thirds of the elements) have been formed in such minute amounts would be very much more

striking to us if we lived on an earth with a perfectly uniform composition. On such an earth, formed without any segregation, it is probable that almost none of these elements would have been discovered. Quite certainly such elements as gold, silver, iodine and arsenic would not be known, and copper, lead, zinc and tin, if known at all, would be in the form of extremely small specimens.

In this connection it may be remembered that *the earth has the highest density of any of the planets.* The data given in Table V show that in the meteorites, which vary in density from about 2.5 for the lightest stone, to more than eight for the heaviest iron meteorites, *the increase in density is not brought about by an increase in the abundance of what have been defined as the heavy atoms, but only by a shift in the relative abundance of the light atoms.* Thus in the less dense stone meteorites, the average atomic percentage of oxygen, atomic weight 16, is 54.7 per cent., while that of iron, atomic weight 55.84, is 10.6 per cent. In the more dense iron meteorites, on the other hand, the percentage of oxygen is practically negligible, while that of iron has risen to 90.6 per cent.[5] A study of the densities of the elements and their compounds shows that the abundance of the elements does not seem to be related to this property. In fact the only apparent relation is to the atomic number, which indicates that the abundance relations are the results of evolution, that is of the factors involved in the formation and disintegration of the atoms.

Since the hydrogen-helium theory was first announced it has been pointed out by Norris F. Hall that both the *isotopic complexity*, and the *number of predominant radiation* of the radioactive elements show a sharp alternation with increasing atomic number, and that this alternation is strictly in accord with the general hydrogen-helium theory of atomic structure. The variation of these properties is illustrated in Figure 4 and it will be seen that the general form of these figures is the same as that of Figures 2 and 3 which represent the abundance of the elements.

ENERGY CHANGES IN THE FORMATION OR DISINTEGRATION OF ATOMS.

When one gram of uranium disintegrates and finally changes into lead, the amount of heat liberated is about four billion

[5]For nickel, atomic weight 58.68, it is 8.5 per cent.

calories, or about five hundred thousand times the amount of heat liberated when the same weight of coal is burned. If the hydrogen which is known to us is not a mixture, then, if it could be made to react with itself to form helium, the amount of energy liberated would be twenty million times that produced by burning the same amount of coal, or one *pound* of hydrogen is in this sense equivalent to one thousand *tons* of coal, with a value

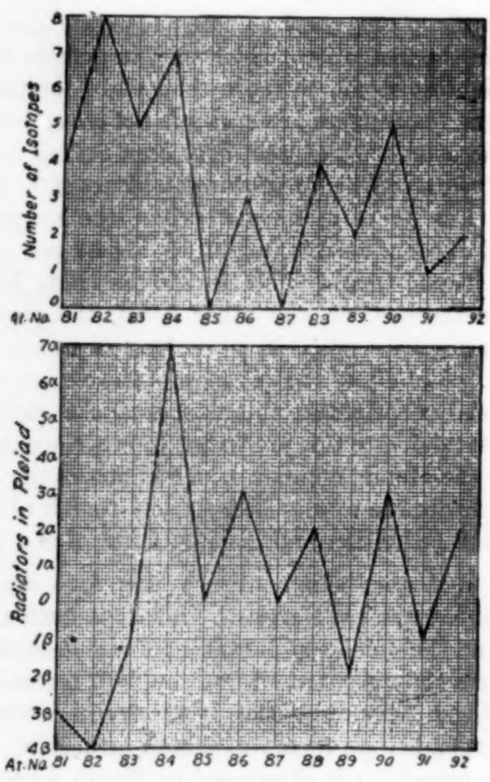


FIG. 4.

of about $10,000. The difficulty at present in the way of the utilization of these enormous amounts of energy is that the first change mentioned above takes many billion years for its completion, and no one, as yet, has any idea as to a method by which the second change may be brought about. As has been stated by Sir Oliver Lodge, the discovery of a method of unlocking these enormous stores of intra-atomic energy, will bring with it enormous possibilities, either for good or for evil.

THE STRUCTURE OF THE HELIUM AND META-HYDROGEN NUCLEI.

The material which has thus far been presented in this paper has a very solid basis in the facts upon which it is based. It may be of interest to speculate as to the structure of the helium nucleus, which is built up from four positive electrons and two negative electrons. The electromagnetic theory of Lorentz, as has been stated by Rutherford, leads to the somewhat remarkable, but easily understood result, that since the positive electron is about 1835 times *heavier* than the negative electron, the radius of the positive electron is 1835 times *smaller*. This is based on the idea that mass is proportional to energy, and the smaller a charged particle is, the greater is the energy due to its charge, since the charge repels itself.

While the more ordinary idea is that both the positive and negative electrons are spheres, they may be discs or rings, or may, of course, have some other form. The model which I would like to suggest for the helium nucleus, or alpha particle, may be made as follows: cut out a disc of thick glass or very heavy cardboard of any desired diameter, say 4 inches. Put this on a horizontal surface, and near its edge put four small discs of metal, say 1-4 inch in diameter, these four metal discs being put at the corners of a square. Above these lay a second disc of glass exactly similar to the first. Seal each glass disc to every metal disc. If it is now imagined that the metal discs are heavy, but the glass discs have almost no mass, this gives a good model for a possible structure of the helium nucleus. It will be noted that in this model the negative electrons (glass discs) do not come closely in contact, and the positive electrons are far apart, but the positive electron comes in close contact with the negative, thus giving an explanation of the loss of mass due to the approach of the positive to the negative electrons, which attract each other.

The meta-hydrogen nucleus may be assumed to have the same structure as the alpha particle, except that, since there are only three positive electrons, they are arranged at the corners of a triangle. The nucleus of the lithium atom may possibly be composed of one of each of the above particles, alpha and nu, and may be assumed to have its seven positive electrons arranged at the corners of two squares which have one corner in common, with two negative electrons above, and two below, one over, and one below each square. It is possible that the lithium nucleus and not the nu group is the lightest particle of odd positive charge. In this case the lithium nucleus would

probably have a different structure. It is of interest in this connection that while one alpha and one nu particle unite to form the nucleus of the lithium atom, two alpha particles will not unite, but three, four, five, six, seven, eight, or ten alpha particles will unite, which suggests that the nuclei of complex atoms are built up of rings or shells of alpha particles, together with any added nu or mu particles. Those who wish to study the details of this system, may refer to the *Physical Review*, February, 1920, or to any or all of the following references:

Journal of the American Chemical Society, 37, 1367-1421, 1915; 38, 186-214, 1916; 39, 856-879, 1917; 41, 970-992, 1919. *Phil. Mag.*, 30, 723-734, 1915. *Science*, N. S., 46, 419-427, 443-448, 1917. *Proc. National Academy of Sciences*, 1, 276, 1915; 2, 216-224, 1916.

THE DISINTEGRATION OF NITROGEN.

By R. W. Millar,

Division of Physical Chemistry, University of Illinois.

[Editor's Note: Great scientific interest has been aroused by a series of experiments performed by Professor Rutherford, because of their bearing on certain important scientific conceptions. If the conclusions reached are verified by later experiments, then this is the first time that one element has been changed to others by an external force. This may necessitate some fundamental changes in our elementary theories and opens vast possibilities for the transmutation of elements. The new notion concerning the law of inverse squares is likewise of great importance in physics and physical chemistry. For more complete information the reader is referred to Rutherford's article in the Philosophical Magazine, June, 1919, or the review in Science *50* 467, Nov., 1919.]

When a disc of metal is exposed to radium emanation, or niton, under certain conditions it becomes coated with a layer of solid radium C which gives off α and β particles. We are here concerned with the α particles, the velocity of which is 19,000 kilometers per second, which is sufficient to give them a range in air at atmospheric pressure of 7 centimeters.

During the greater part of this range the velocities of all the α particles are constant and equal, but in the last centimeter they lose practically all their velocity and are deflected by the molecules of air. These α particles, each with a mass of four with respect to the hydrogen atom, have an enormous kinetic energy. Rutherford calculates that one gram of matter moving with their velocity has the same kinetic energy as 10,000 tons moving at a speed of 1 kilometer per second. Their energy is so great that when a particle impinges upon a crystal of zinc sulphide a flash of light is seen. The particles will pass through thin plates of aluminum or gold until the stopping power of these plates is equivalent to that of 7 centimeters of air. Thus a thin plate of metal may be substituted for any part of the 7 centimeters of air in order to decrease the actual speed and range of the α particles.

Rutherford replaced air with hydrogen and found that the number of scintillations observed on the screen remained constant up to 19 centimeters from the source of α particles, and then fell off rapidly, becoming zero at a range of 28 centimeters. The β rays were deflected from the field by means of an electromagnet. By interposing absorption plates in order to prevent any α particles from reaching the zinc sulphide screen, Rutherford was able to determine the number of hydrogen atoms at each range. Then, by placing foils of metal in contact with the source of α particles he was able to secure α particles and to observe the number of scintillations produced by them.

Marsden had shown that hydrogen atoms could produce scintillations on a zinc sulphide screen similar to those produced by α particles. The hydrogen atoms were evidently set in rapid motion by collision with the α particles, and being lighter, were given a greater speed and consequently greater range. However, the speeds and the distribution of the swift hydrogen atoms were far different from those calculated by Rutherford on the theory that the α particles were point charges. The only method of accounting for the number and distribution of the swift hydrogen atoms was by assuming that the α particles approached within $2.4\text{x}10^{-13}$ centimeters of the center of the hydrogen and that at this small distance, which is less than the diameter of the electron, viz., $3.6\text{x}10^{-13}$, the law of inverse squares holds.

Marsden and Rutherford found that a small number of scintillations were always observed up to 28 centimeters in air. These appeared in spite of all precautions to remove hydrogen from the materials used. Whether they are a product of the disintegration of radium C or from some other source, could not be ascertained. Their number was the same when the α particles passed through either oxygen or carbon dioxide.

When nitrogen or oxygen is used in place of hydrogen, swift atoms are observed up to a range of about 9 centimeters. It is seen that, owing to their greater mass, these atoms are given much smaller velocity than hydrogen atoms. A few hydrogen atoms of the unknown origin appear which have a range of 28 centimeters. Pure, dry nitrogen, however, gives a much greater number of swift atoms of range 28 centimeters than oxygen or carbon dioxide, or air. Air, however, gives a larger number than oxygen or carbon dioxide.

These swift atoms appear to have the same mass, charge, and

velocity as the swift hydrogen atoms obtained by the collisions of α particles with hydrogen as shown by measurements of e/m for swift atoms from the three sources: hydrogen, nitrogen, and the source or hydrogen occluded in the materials used. Furthermore, nitrogen, when bombarded by α particles, appears to be the source of hydrogen atoms. The number thus obtained is, however, comparatively small.

Rutherford has given evidence of three possibilities.

1. Radium C may radiate H atoms as well as α and β rays.
2. The law of inverse squares probably holds at distances smaller than the diameter of the electron.
3. Nitrogen may be disintegrated by α particles giving hydrogen as a product.

ILLUSTRATION OF MOLECULAR MOTION.

By J. Norman Taylor,

Washington Preparatory School, Y. M. C. A., Washington, D. C.

A proper appreciation by secondary school students of the "habits of Nature," which are expressed in abstract statements called "laws" because of their unfailing truth, is more readily brought about if concrete examples are given to illustrate their application.

For instance, it is very difficult for an immature mind to appreciate the modern theory of the composition of matter. When it is said that matter is not continuous but is of its own nature discrete, i. e., composed of unit particles known as molecules, it is difficult for the student to grasp the full meaning of this abstract statement. His idea regarding the behavior of these molecules is also a vague one. It is very hard for him to visualize the movements of particles infinitely small, contained in a transparent vessel, and themselves invisible. If he can perform an experiment which will illustrate to him how molecules move very rapidly in straight lines until they collide with each other or come into contact with the walls of the containing vessel, then he will be able to perceive that molecules are elastic. He will also be able to accept the postulate that the interstices between the molecules must of necessity be larger than the molecules themselves. Furthermore, if a student is enabled, through an appropriate illustration, to understand that when heat is applied to the system the molecules move much more rapidly, then he will be in a much better position to take up the

study of the gas laws. He will also be enabled to understand more thoroughly the kinetic theory of gases which is based upon all of these considerations.

A device illustrating the assumed behavior of molecules, and described by E. R. Stoekel in "Science," Vol. XLVIII, No. 1245, has been employed by the writer in assisting chemistry students to a better understanding of the molecular theory. It consists essentially of a hard glass tube about ten inches in length and one inch in diameter containing a pool of mercury which supports a small quantity of finely crushed material. In using this form of apparatus in the Association School laboratory it has been found that particles of cobalt glass of about twenty mesh are very satisfactory. After preparing the tube, as here indicated, it is evacuated so that a pressure of less than a millimeter obtains and is then sealed from the pump. During the subsequent demonstration the tube may be held in the hand without inconvenience but may, if desired, be clamped to a standard.

Upon the application of heat to the mercury pool and upon gradually increasing the temperature, the particles on the surface "were carried away by the evaporating mercury" and moved about in the upper part of the tube in much the same manner as gas molecules are presumed to behave.

After completion of the demonstration by the instructor each student may be allowed to verify the experiment. The use of this simple device excellently enables the student to form a clear mental picture of how the very small gas molecules are supposed to be in constant motion and how the speed of this motion is altered by changes in temperature.

POTASH DEPOSITS IN SPAIN.

Potash deposits were discovered a few years ago in the Province of Barcelona, Spain, near the villages of Suria and Cardona. They consist of irregular beds of carnallite and sylvinite interbedded with rock salt. Explorations to a depth of several hundred feet show that in the Suria district potash beds occupy an area of not less than 75 acres and occur at depths from 125 to 200 feet. The average combined thickness of the carnallite beds is estimated to be about 56 feet, and of the sylvinite about 13 feet. The Cardona upper beds are interbedded with gypsum and clay, but the lower beds contain nearly pure white salt, which is that principally mined. After the discovery of potash at Suria, these Cardona beds were searched for potash, and nearly pure sylvite was found. Estimates for the area prospected place the quantity of carnallite at 2,550,000 tons and of sylvinite at 1,150,000 tons.—[*U. S. Geological Survey.*

THE PLACE OF GENERAL SCIENCE IN THE HIGH SCHOOL.[1]

BY JOHN CALVIN HANNA

State Supervisor of High Schools, Springfield, Ill.

(*Appearing on the program as "General Science in the High School of Tomorrow."*)

The title of the paper suggests these questions:

1. Should general science have a place in the high school?
2. Should it be a prescribed or an elective study?
3. How long a course should it be?
4. How should it be organized?
5. Where in the course should it be placed?

These questions are involved each with the others and it is difficult to answer them independently.

The answers to these, in the opinion of the writer, may be concisely expressed thus:

1. General science has a legitimate place in the high school—a place won after long experimentation.
2. It should, if offered at all, be prescribed for all students—whether in large or in small schools and whatever kind of a course is to be taken by the pupil, as well as whatever his plans are after completing a high school course.
3. The course in general science should be a full time one-year course, with at least one double time period a week given to laboratory work performed by the pupils themselves.
4. It should be organized with special reference to the pupils' degree of maturity, as a basis for further studies and as a real introduction to the whole field of natural science. It should, therefore, be organized with a sound logical foundation and conducted with due reference to the natural interest of youth in concrete illustrations of principles and familiar phenomena produced by the operation of nature's laws.
5. It should come first—before the study of special fields of natural science and before the study of applied science in vocational fields. That is, it should be taken in the ninth grade.

If in some schools it shall prove possible and wise to reorganize the work of the seventh and eighth grades in accordance with the sanest and most consistent propositions of the advocates of a junior high school,then it might be taken in the junior high school. If so, it ought to be longer than a one-year course—possibly to run through the seventh and eighth grades.

Its position, under these circumstances, might be determined at the same time and in the same way as might the position of elementary manual training.

The writer was led to these opinions on those five points first, by a vivid impression that the ordinary approach to science study in high schools was false pedagogically, as well as most unsatisfactory in its results.

That impression was made first some twenty-six years ago

[1]Read before the General Science Section of the Central Association of Science and Mathematics Teachers, held in Chicago, Nov. 29, 1919.

or more and has been confirmed by studies ever since that time. I have no doubt that a similar impression was made upon the minds of many teachers in high schools, both upon those who were teachers of botany, zoology, physiology, chemistry, physics, and physical geography, and upon other teachers as well who, like myself, were teachers in other fields, and especially also upon thoughtful administrative officers, principals, and superintendents, and also upon some college professors.

Whether the impression was developed as early as that in the minds of the masters of pedagogical theory, I do not know. So far as I have read or heard, none of them published anything on the subject until long after that.

Most of those, I believe, who began to develop those ideas were practically *forced* to it by the actual conditions which prevailed.

After the period of "natural philosophy" studied from a book only—a period and a plan which in the hands of an enthusiastic teacher had the result, no doubt, of stirring interest in science studies in the minds of many a youth; after the period next following, viz., the period of the fourteen weeks text books—books which unquestionably had their place in the progressive movement in regard to science studies, then came the period of highly differentiated and more or less perfectly developed laboratory courses in several of the subdivisions of natural science: physics, chemistry, botany, zoology (with the incidental struggle between these two and a course labeled "biology"), human physiology and the earth science study commonly labeled physical geography or physiography (sometines degenerating, or if that be too harsh a word, narrowing itself to a course in the study of erosion, pure and simple).

The specialists captured the field. Fresh from special training in the universities, burning with enthusiasm and a fierce loyalty each to his own special field, these young teachers, many of them at least—in the long and painful struggle for precedence among the special subdivisions of science study—some at least of these teachers developed into narrow and bitter partisans, so that the war between science specialists soon overshadowed the ancient and chronic war between the advocates of science studies and the advocates of the humanities—causing, no doubt, many a chuckle of sinful glee among the defenders of the humanities who, very naturally, and very foolishly, rejoiced at this division in the ranks of their opponents.

In the meantime, some actual trials were being conducted of a plan whereby not any one but all of the different fields of science study should be included—even if in brief and elementary fashion—still, should be included honestly and in good scientific spirit in an introductory course of elementary science.

This was not yet labeled "General Science." It was wrought out painfully with open mind by teachers who sought not each to glorify his own special field (for of course all of them had a special predilection for some such special field), but whose whole study and aim was to find out what kind of a course best suited the age of the pupil entering a real high school, and best prepared him for what he was to do afterwards—(first) in the later special science courses almost universally taught, (second) in taking up later such special applied courses as agriculture and household science, (third) in actual preparation for the duties of subsequent years, whether a part of those years was to be spent in college or not.

I am reciting especially the story of the Oak Park experiment, but I am confident that somewhat similar experiments were conducted in other schools in the earliest years of the present century. I am informed that President Hessler, of James Millikin University, has collected a considerable amount of material concerning this earliest pioneering, and I hope its publication may be possible in some form that will help to clear some of the problems for such a body as this.

The writer was fortunate in being one of those connected in a capacity of general supervision and encouragement with those earlier studies and experiments and the results amply justified the efforts of those faithful, modest and painstaking teachers, efforts conducted through a term of nine years with classes of 150 to 350 new students each year and with laboratories kept busy all day, but *without any printed textbook* (for none had yet been printed).

Certain conclusions were reached by these studies:

(1) Such a course is possible and is greatly needed.

(2) It must in its formation recognize intelligently the logical classification of science phenomena under the headings which label the ordinary special sciences.

(3) These lines of demarcation need not be made so conspicuous as to disturb the even tenor of the pupil's development in the scientific method of study nor his increasing recognition of the great underlying truth that nature's laws, many of them, are manifest everywhere and the phenomena which obey these laws are almost inextricably tangled.

(4) Such a course will rouse and maintain the interest of all pupils if properly presented. This is an overwhelming statement and yet is based upon a sufficient number of examples to be perfectly safe.

(5) It must include the actual performance of definite and carefully selected experiments by the pupil himself under the guidance of the teacher, with a constant training of his power to record phenomena and draw conclusions. Such a training is of value, not only in future natural science studies. It is of vital importance in all studies and applications of social sciences—in history and civics, in the political, economic and sociological problems of life, in living as a safe and helpful citizen.

(6) The practical questions incident to the putting of such a course generally into successful operation are important, such as the supply of teachers fitted for this work and the cost of equipping a special laboratory.

Some opponents of the whole movement have based their opposition on the fact that the field of natural science has become too vast for any one teacher to master it all, and that as a consequence it is folly to expect or to pretend that we can have properly fitted teachers of "general science."

This objection falls to the ground when we bear in mind that not one high school in five in this state is large enough to employ a separate teacher for each of even three or four sciences.

Not more than about one hundred high schools in Illinois have a senior class so large that even two sections in physics are formed, and only perhaps fifty or sixty high schools have a sufficient enrollment to require the formation of three physics sections—the number that would be necessary to warrant the employment of a physics teacher who teaches no other subject.

The fact is that in ninety per cent of the high schools of the state, whether any so-called general science courses are taught or not, the same teacher conducts classes in two or more sciences and does the work as well as it is done in other departments of the school—history, mathematics, language. Few, if any, high school teachers pretend to be *masters* even of one field of study, but a very large proportion of them do fairly acceptable work in more than one field, and especially in fields as nearly related as are those of the different science subdivisions.

The other practical question, the equipping of a laboratory, is easily answered. Most small high schools (and most high schools *are* small—enrolling less than 100 pupils) do very well with one laboratory room for all science classes.

Many of the smallest schools, even those with only ninth and tenth grades maintained, are now doing good, honest, helpful work with some laboratory experimentation conducted by the pupils themselves at a table in the same room where other classes are conducted—the *only* room used for all high school work in that particular school.

It would be an enlightening experience for some high school principals and teachers if they could accompany Mr. Thrasher

or myself in our visits to the recognized two year high schools of the state—schools where a total enrollment of twenty is something for the principal to point to with pride, and to see the work accomplished there in all subjects—even in general science.

Others of these schools are not so good, but there is a steady improvement. Many pupils who have had two years in such little schools are now making creditable records as eleventh and twelfth grade pupils in strong four year high schools and others still are winning out in college or university.

This presentation of the results of actual trial goes far, it seems to me, to determine the possibility and value of such a course not only, but also to furnish an answer to the fifth question appearing at the head of this paper, with the statement that experience proves that this course not only should precede all other special science studies, but should precede the domestic science and agriculture courses in the high school. Furthermore, the experiments justify the conviction that the course should be prescribed for all pupils and not allowed merely as an elective. The peculiar value of the training which it gives is needed by every boy and girl facing youth and its privileges.

In the requirements for recognized high schools in Illinois it is included that the course in general science, if taken at all, must be a year course with at least one weekly double time laboratory exercise and that it shall include a sufficient amount of the study of human physiology to satisfy the statutory requirement of forty lessons in that subject and that it shall be required of all pupils in the ninth grade. In small schools with a small enrollment in the ninth and tenth grades, respectively, making possible the uniting of those two classes in one subject, the general science may be offered in alternate years, the two classes uniting in one year in general science, and in the alternate year in ancient history or in certain other subjects exactly set forth in published circulars.

The practical question arises as to what entrance credit should be given by colleges and universities for such a course. Many colleges and universities even now allow it one full entrance unit and the tendency has been to agree to such allowance whenever university authorities have actually inspected the working of such a course in a school where it is well taught.

There is in some quarters, notably the University of Illinois, a tendency to be suspicious of the value of such a course—analogous no doubt to the long continued hesitancy of universities

to allow any entrance credit for manual training or domestic science—a conservatism based originally no doubt on sensible considerations, but seeming to some impatient young principals and superintendents a hesitancy too long protracted and unfair. These principals and superintendents have found such a course of value, have seen it produce good results and they feel that, in the words of Stephen A. Forbes uttered twenty years ago, "It is the business of the university to learn what the high schools can do and do well and then to accept that for entrance credit."

This paper is not the place for discussion of the university's policy. Let us hope that the decision to accept general science for entrance credit under proper limitations and restrictions may not be much longer delayed.

In the meantime, I have not hesitated in consultation with those school authorities where there was a strong desire to conduct courses in general science and a chafing under the fact that this would cut out one entrance credit at the university for pupils who offer it, to call their attention to the fact that any college or university requires fifteen units for admission without condition from an accredited school, and that all recognized high schools must require sixteen units for graduation. The sixteenth unit may be general science. Of course, this makes it necessary for the school to secure full unit-for-unit credit for the other subjects in its course, and this is something possible for a school really deserving the full accrediting relation.

Usually the hitch is in the English. Perhaps thirty or forty per cent of the public high schools on the university's accredited list are giving four years to English and receiving only three or three and one half units for that work.

Some of those schools, no doubt, could so strengthen their English curriculums as to conform to the university's requirement for securing four units for four years of English, and in such cases, I have, when asked for advice, given the advice, "Strengthen your English, win four units, and hold to your general science if you really want it."

There are now, or were in August 1919, in Illinois 540 recognized four-year high schools (of which 75 have probationary recognition), 94 recognized three-year high schools, and 141 recognized two-year high schools, most of these two latter groups having probationary recognition.

A careful examination of the records for 1918-19 shows that 144 of these 540 four-year high schools offer a course called

general science and most of these require it of all pupils in the ninth grade. There are also about 50 or 60 of the two-year and three-year high schools offering general science; nearly 26 per cent in all.

Nearly all of the other high schools in 1918-19 required of their pupils in the ninth grade instead of a general science course a unit composed of two half units, one of which is physiology and the other physical geography or botany or zoology or elementary civics or commercial geography.

These pupils in those schools *not* teaching general science, in a very large number of cases, are put through two half units—one of physiology and one of physical geography, and in a considerable number at least of the smaller schools where no general science is taught, this first year's work is followed by a half year each of zoology and botany.

The question of course arises whether those half units, three in a biological field and one of earth science, constitute a safer, more helpful and sounder introduction to the later activities of the youth, both in and out of school, than does the year course of general science. Even if there be objection, one might say, to these brief science courses—each of one half year—surely that objection would apply with increased force to a year unit in which the whole field of natural science is attempted to be covered.

There is the crux of the whole question. A good general science course is not an attempt to "cover," to go over, to explore the whole field of natural science, It is not a grand lightning tour; it is a bird's eye view, and as such should precede any particular toad's eye view.

It is for the purpose of enlightening the mind and rousing the interest of the pupil to the great truth that *law* runs through the natural world, and that not only in a detailed study of microtomic sections of a leaf stem or a crayfish's structure is there manifest the operation of law, but that law governs mechanics, sound, light, heat, electricity, chemical change, animal and plant life and the phenomena of geography and meteorology.

It is not necessary thoroughly to explore nor to cover all these fields in order to establsh forever in the adolescent mind this great truth.

A few carefully selected experiments in each of these fields, skillfully utilized by a careful instructor with his eye set upon the real aim to be reached, and with constant reference to the

fundamental principles thus established will accomplish the *result*, the recognition of the imperial sway of nature's law, and will rouse the interest and ambition of many a youth to further studies in some one of the special fields toward which in this preliminary survey the bird's eye has been turned.

It will, of course, serve also the other purpose, as important in the pedagogy of adolescence, the training of the mind to observe phenomena, record evidence and draw conclusions from evidence only—a training of the utmost value in preparing for citizenship—a training to be accomplished for practical purposes not only through the theorems and problems of geometry, though these are almost essential to making the fiber firm, but also through a study of law's operations in the phenomena of natural science, and indeed of the social sciences, where, of course, is shown the special application of this training to many of the problems of citizenship.

Since all youths are to be citizens, and therefore need such training, since the preliminary view is needful for the growing mind just beginning to think for itself, and since experimentation has demonstrated the possibility of doing this thing well through this medium in one year and not in less, and since more time can not be spared and, speaking practically, is not needed, it is to be concluded that general science should be taught for one year and to all pupils in the ninth grade, with the possible exception as to junior high schools referred to above.

One item in this conclusion perhaps may still be questioned: "And since more time for such a course can not be spared and speaking practically is not needed."

Why not give *two* years to this general view before taking up special sciences?

The answer is found in the stern fact that this question, and all questions of curriculum are not going to be decided by specialists, but by pupils, parents, boards of education, and by superintendents and principals, each of whom has the task of working out a plan whereby the best use of four years will be made by actual living boys and girls, the selection of sixteen units from the vast number classified as proper for use in high schools.

The general list of such units approved for recognized high schools foots up 48½, and the addition of other units actually approved and in operation in some of the larger schools would bring the number probably to sixty units, allowable in high school courses—out of which sixteen are to be selected for any individual

boy or girl. Some one may say that is an easy matter, agree upon a brief list of absolutely prescribed units, say six or eight of the total of sixteen, and make electives of all the rest.

This may work in a very few large schools, but in the overwhelming majority of our high schools the range of electives is and always must be greatly limited. No school in Illinois offers the whole range. Most schools have no electives at all or a very limited range.

An examination of the published list of 540 recognized four-year high schools shows that out of the first 287 in the alphabetical list (almost one half) and including the twenty-two large Chicago high schools, 220 of these schools have not more than ten teachers—that is, more than 76 per cent of these schools can not offer a very wide range of electives.

Furthermore, 156 of them, that is 54 per cent, have not even five teachers and half of them have only three teachers, which means practically no electives but a fixed course for all pupils.

Therefore, the elective loophole will not show the way out for pupils in more than half the four-year high schools of the state and, of course, nothing of the kind is possible in any of the three-year or two-year high schools which, in themselves, constitute more than one third of all the recognized high schools in Illinois. And these small high schools are not to be ignored, not to be despised. Whoever does so merely advertises his own ignorance. The material in them is just as good. Much of the instruction in them is good and many of our best college and university students started in such schools.

Three fourths of all high schools are "small" high schools, with an enrollment of less than one hundred. One fourth of the whole hundred thousand high school pupils in Illinois are in such small schools. The problem thus narrows itself to what is the best selection of sixteen units to be laid down for all boys and girls—omitting a few hundreds or thousands of exceptional ones in exceptional environment.

Suppose a joint high commission were charged with the duty of framing such a course, and suppose such commission to be absolutely unbiased and to have an eye single to one aim. Then suppose this commission should ask each group or section of specialists for the number and title of the minimum list of units in that particular field which, in their judgment, should be included in such a total list of sixteen units. How do you think they would come out?

The English people would insist on four units—and some think that limit is all too short.

The mathematics people will call for three units.

The history and social science section people will call for four units.

The language specialists demand four units at the very least and are aggrieved if this be cut to three or two.

The science specialist can see nothing less than four units and even then *somebody* is crowded out.

The vocational and prevocational demands will not down. Their claims run from four down to two units.

And then there are the later demands for economics, physical training the arts; and I came near forgetting the claims of the commercial branches.

Here they are—English, mathematics, ancient languages, modern languages, history and social science, natural science, vocational, commercial, artistic and physical—at least nine distinct fields, each of which includes two or more subdivisions.

How can their claims be ignored? How can any of them claim four units?

Let me go through the motions of acting as if I were that commission or had the deciding vote in its deliberations.

Only one of these can with reasonableness demand four units required of all pupils in all courses—and that is English. If we other specialists are reasonable, we are all going to vote for that list of four.

Mathematics must be satisfied with two, one each of algebra and geometry, with the content of each trimmed and readjusted to fit the modern world.

History and the social sciences must be given three units and this is little enough in view of the ghosts that have arisen lately to confront us.

As to languages, ancient and modern, I will leave them all out of the prescribed list, if I can get all the other specialists to be half as reasonable. And yet I have strong faith in the value of Latin properly taught and of the practical usefulness of a working knowledge of some of the modern languages.

There must be at least one unit of hand work for all pupils; otherwise we ignore the most outstanding fact in youthful psychology and physiology.

One unit for music and drawing, or at least one of them, if it were a possibility, is sadly needed. We need to educate for that one-third of every twenty-four hour day which is devoted to recreation—that one-third of the day within which is found all the crime and vice.

There is almost as serious a need for some universal instruction in practical accounts, and physical training must be looked

after. Suppose each of these two has a half unit; then where do we stand?

English	4 units
Mathematics	2 units
History and social science	3 units
Hand work	1 unit
The arts	1 unit
Commercial and physical	1 unit
A total of	12 units, leaving

four units for other groups. Natural science should be given two of these as prescribed units and two only, and the other two units should be free for choice on the part of the individual pupil in a school with a range of electives and for the school itself where a range of electives is impossible.

Such a distribution of units, it seems to me, would commend itself to an unprejudiced and intelligent being from another planet, making a complete study of our needs. Now, if this is fair, we are going to make a selection of two science units which *must be taken by all pupils* in all high school courses. What shall they be?

My own feeling is that the first should be what we are calling general science, organized as I have suggested, and the second should be what the pupil (or the community) calls for—but that it should be a *special science* unit thoroughly studied for a year.

The pupil should have a chance to carry through one full year's work in one special field of science after going through his preliminary year of general science.

It may be a biological unit, a unit of chemistry or of physics or of earth science—for many pupils, in my opinion, it should be a unit of advanced geography—but that is another story.

I close, not hoping to settle these great questions, but to have contributed a little to lead us who are or have been specialists to rise up high enough to see how broad is the world of education and how necessary it is to recognize the value of other wholly separate fields in the arrangement of the high school course—a work finally to be done by the administrative groups of educational experts.

TALK ON LOGARITHMS AND SLIDE RULES.[1]

By Florian Cajori,

University of California.

During July 24-27, 1914, there was held at Edinburgh, in Scotland, an international gathering of mathematicians, to celebrate the three hundredth anniversary of a great scientific event—the invention of logarithms.

Besides British men of science, there were present French, German, American, Russian, and Turkish scientists. That the celebration of an invention in mathematics should be the occasion of an international gathering seemed to indicate that the world had indeed risen far above ordinary intellectual levels and had reached a disposition to friendly intercourse and a spirit of real comity.

Little did we know what was brewing in the dark and hidden recesses of political intrigue. The only visible clouds were some minor and local social struggles. One of the meetings for the entertainment of the foreign guests was to be held in M'Ewan Hall of the University of Edinburgh,—a hall noted for its elaborate and artistic decorations. The place of meeting was finally changed to an unpretentious club house, to prevent the possibility of injury to the decorations by the militant suffragettes. The mathematical program was carried through without a disturbance of any sort. The suffragettes abstained; all foreign representatives met in friendly intercourse. The town of Edinburgh gave itself up to the celebration of the achievements of John Napier, who, next to Sir Walter Scott, was proclaimed the greatest of Scotsmen.

At the scientific meetings, papers of historical interest relating to logarithms and methods of computation were read. Mathematical instruments of all sorts and mathematical models were exhibited. It was made plain that the logarithms published by John Napier, in 1614, were not what mathematicians unfamiliar with the history of their science often think they were. Napier's logarithms were not to the base e ($= 2.718 \cdot \cdot \cdot \cdot \cdot$), nor to the base e^{-1}. In fact his logarithms do not admit of being explained on the concept of any base. If a positive number b that is larger than unity is selected as a base, then 0 must be the logarithm of 1. In Napier's logarithms this relation did not hold. His system rested on a different foundation.

[1]A talk given before the undergraduate Mathematics Club of the University of California, March 10, 1920.

In his system, 0 was the logarithm of 10^7. His tables were primarily constructed for computations involving the *sine* function in trigonometry. He took $sin 90°$ equal to the radius and the radius equal to 10^7. It seemed easiest to him to arrange his system so that the logarithm of $sin 90°$ would be zero. Hence $\log 10^7 = 0$. In his system the logarithm of a product was not equal to the sum of the logarithms of the factors, but in solving proportions, his results were theoretically accurate.

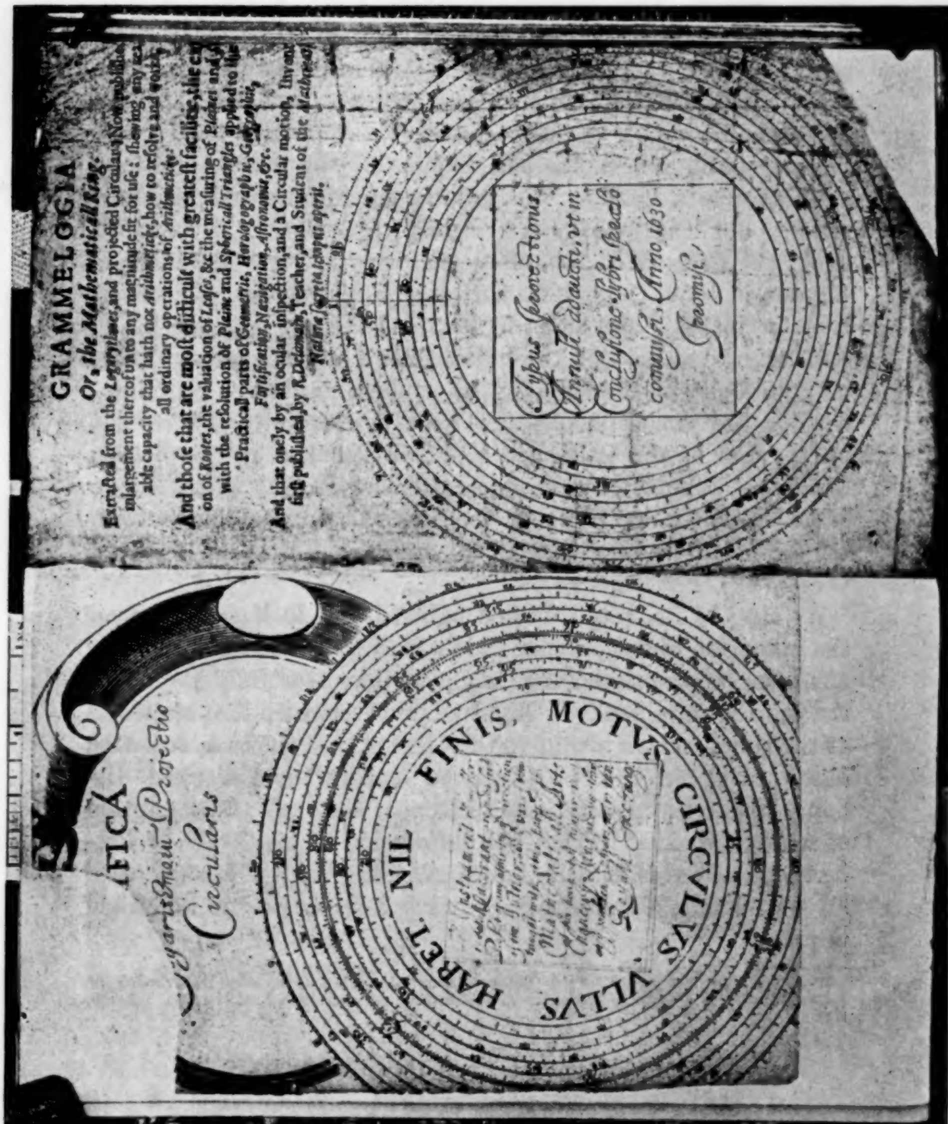
Our common logarithms, to the base 10, were agreed upon at a meeting of Henry Briggs with John Napier; Briggs had gone from London to Scotland to visit Napier. Which of the two men deserves the chief credit for adjusting logarithms to the base 10 is not known. At the congress of 1914 in Edinburgh, a Scotsman argued that the main credit is due to Napier; an Englishman argued in favor of Briggs. This divergence of opinion was probably not due to sectional bias though unfortunately, national predilection is often evident in the history of science.

One of the mathematicians from Cambridge, Dr. J. W. L. Glaisher, was able to contribute a new historical fact on logarithms. He pointed out[2] that the natural logarithms, to the base $e(= 2.718 \cdot \cdot \cdot \cdot \cdot)$ came in earlier than was formerly supposed. They are found in a publication which appeared only four years after Napier's *Descriptio* of 1614. The second (1618) edition of Edward Wright's translation of Napier's *Descriptio* into English contains an anonymous *Appendix* describing a process of interpolation with the aid of a small table containing the logarithms of 72 sines. The latter are natural logarithms with the decimal point omitted. Thus, log 10 = 2302584. This *Appendix* which contains the earliest natural logarithms known to history is probably from the pen of the English algebraist, William Oughtred.

Seldom is a great discovery or invention the uncontested output of one mind. For the invention of the telescope, pendulum clock, and electric telegraph there are in each case several competitors. In the case of logarithms, John Napier must divide the honor of invention with Joost Bürgi, a Swiss clockmaker and mathematician. As Bürgi published his tables in 1620, six years after Napier's, the priority lies clearly with the Scotsman. So rare is Bürgi's book on logarithms that none of the British and American mathematicians attending the celebration had previously seen it. The copy exhibited in Edinburgh had been

[2] *Quarterly Journal of Pure and Applied Mathematics*, Vol. 46, 1915, p. 145.

borrowed from the library of the city of Dantzig, at that time located in Germany.



The Title Page of the Earliest Book Published on the Slide Rule.

There were exhibited at the University of Edinburgh, also, several designs of logarithmic slide rules. New revelations have

been made in recent years on the early history of the slide rule. Until recently altogether erroneous data were current regarding its invention. That Edmund Wingate, a writer of arithmetics, invented such an instrument is now definitely disproved. There were two rival claimants for the invention of the circular slide rule, namely, William Oughtred, whom we have mentioned as the probable author of the earliest table of natural logarithms, and one of his pupils, Richard Delamain, who was a teacher of mathematics in London. Oughtred's invention antedates Delamain's, but Delamain was the first to publish.[3] Delamain's book, the *Grammelogia or Mathematical Ring*, first appeared in London in 1630; Oughtred's *Circles of Proportion* were issued at London, in 1632, and again in 1633. As Delamain's text is the earliest printed book on the slide rule and as it has only recently been brought to the attention of modern readers, a reproduction of the two title pages will be of interest. We exhibit photograph of the title-page of a copy that is in the British Museum. The book is very rare. The present writer knows of no copies except those in the British Museum in London, the Bodleian Library at Oxford, and the Library of the University of Cambridge.

As already indicated, both Oughtred and Delamain claimed the invention of the circular slide rule. Each accused the other of having appropriated the idea. We have carefully gone over the printed statements of Oughtred and Delamain that appeared at the time, and we incline to the opinion that the two men were independent inventors. Oughtred enjoys unchallenged the honor of the invention of the rectilinear slide rule, a description of which appeared in the 1633 impression of his *Circles of Proportion*. He says that he had made models of slide rules with his own hands about eleven years before they were described in print.

---

[3]For details see F. Cajori, "On the History of Gunter's Scale and the Slide Rule during the 17th Century" in *University of California Publications in Mathematics*, Vol. 1, pp. 187-209, 1920.

FRESHMAN COLLEGE MATHEMATICS.

In this discussion we assume that the purpose of freshman mathematics is to give to the student the most valuable mathematical information which he is capable of receiving during that year and to develop in him the power to analyze and understand relations of quantity and space as expressed by,

1. The function.
2. The general algebraic method.
3. Applications of the principles of geometry.
4. The formula and computation.
5. Simple applications of the derivative to problems in maxima and minima, and rates.

What then shall be the content of such a course? Trigonometry and analytics use or refer to approximately one-third of the theorems in plane geometry, algebra is almost entirely independent of geometry as a basis for theory and until recently for its problem material. Thus our present course in college mathematics might be said to have as a basis, twelve topics in algebra, and sixty propositions in geometry. Of the latter not more than forty are fundamental in the theory work. Until recently applied problems in geometry, physics, chemistry, domestic science, or the evaluation of formulas were exceedingly rare, and the vitalizing facts of history and biography were entirely omitted. Theorems and topics needed for foundation work or used for discipline have been treated indiscriminately. Some of these facts are seen more clearly by referring to table VIII which shows the order of growth and development of topics in college algebras beginning with 1796 and covering a period of 122 years. The high water mark in the number of topics, pages and problems given was reached in the period from 1900 to 1910. One text written for beginners treats forty-six topics and uses 8,000 problems.

Two striking changes are seen in our recent texts. First, they omit many of those topics belonging primarily to the high-school course and place light emphasis on topics having doubtful social worth. Second, they place an increasing emphasis on the graph, compound interest, maximum and minimum, probability, evaluation of formulas, historical notes, and timing exercises.

Below we have listed topics of algebra under three heads in order to show the present tendency. First are those topics which have been dropped from the newer texts or almost so. The second classification enumerates the topics that are ap-

parently receiving less attention. The third classification shows the topics that are apparently increasing in importance and which will appear more prominently in all freshman mathematics within a few years.

I. TOPICS IN COLLEGE ALGEBRA WHICH HAVE LARGELY DISAPPEARED FROM THE NEWER COLLEGE TEXTS. (BRIEFER TEXTS.)

1. Geometric construction of equations.
2. Harmonic progression.
3. Geometric proportion.
4. Linear equations of four or more unknowns.
5. The multinomial theorem.
6. Symmetry.
7. Euclidean method of obtaining H. C. F.
8. Comparison as a method of elimination (Simultaneous equations).
9. Indeterminate equation.
10. The problem of the couriers.
11. The problem of the lights.
12. Continued fraction.
13. Reciprocal equations.
14. Recurring series, summation of series.
15. Reversion of series, convergency of series.
16. Differential method of obtaining series.
17. Scales of notation.
18. Theory of numbers.
19. Indeterminate coefficients.
20. Cardan's solution of cubic.
21. Sturm's functions.
22. Newton's method of approximation.
23. Upper and lower limits to roots of equations.
24. Factorial binomial theorem.

II. TOPICS IN COLLEGE ALGEBRA ON WHICH LESS EMPHASIS IS BEING LAID.

1. Involution and evolution.
2. Quadratic surds.
3. Simultaneous equations of three or more unknowns.
4. Factor theorem.
5. Topics belonging to ninth grade algebra.
6. H. C. F. and L. C. M. Sometimes omitted.
7. Powers of polynomials.
8. Progressions, except the infinite geometric series.

9. Cube root.
10. Solution of cubic, except by Horner's Method.
11. Determinants of order higher than three.
12. Binomial theorem.
13. Long, compound, and complex fractions.
14. Partial fractions.
15. Inequalities.
16. Complex numbers.

III. TOPICS IN FRESHMAN MATHEMATICS WHICH ARE BECOMING MORE PROMINENT OR AT LEAST APPEARING.

1. Graph.
2. Compound interest and annuities.
3. The derivative and its applications.
    a. Maximum and minimum
    b. Rates.
4. Probability.
5. Evaluation of formulas.
    a. Geometry.
    b. Physics.
    c. Mechanics.
6. Timing exercises.
7. Biography, historical notes. There seems to be a tendency to include more problems having a social interest.
8. Integration is found in some of the recent texts.

It is doubtful if any course in the curriculum can lay for the student the foundation for the solution of business or economic problems, the interpretation of natural phenomena, the study of science, engineering, statistics, or bring the student into a closer fellowship with more than one hundred of the greatest men and women of all ages together with their method of thinking than can mathematics. Let us endeavor to make it vital in the student life and give it its proper place in the development and education of men and women.

A SUGGESTED COURSE IN FRESHMAN MATHEMATICS.

An attempt is made in the following outline to separate into groups the topics that have considerable claim to treatment in the freshman year.

GROUP A includes the topics which should be given special emphasis. The committee recommends that a three-hour course should include at least all the topics of Group A.

GROUP B includes topics that should receive consideration but not quite the same emphasis or certainty of treatment as

the topics in Group A. Thus a four or five hour course should include the topics of Group B as well as those of Group A.

GROUP C includes topics that are suitable for the capacities of freshmen, and may well be included in a five-hour course. However, they should be regarded as belonging to the superstructure of the course rather than to the topics that should be mastered.

In submitting this outline we have in mind particularly the student who takes only one year of mathematics. The thought is to give him a course that will be useful in reading scientific literature, in solving problems that he is likely to meet and in giving him a notion of the value of mathematics as an idealistic subject.

Students come into our mathematics classes at a time when they are forming their ideals and we should not neglect the ideals of scientific truth as they are involved in mathematics any more than we would the ideals of literature. All of us have heard of the recent attacks upon the place of mathematics in the schools. Our best answer to these attacks lies in studying the conditions and adapting mathematics to the new conditions. In considering the changed conditions, the place of mathematics is so well expressed by R. G. Wells in his *Mankind in the Making* that we submit the following for your consideration:

"The new mathematics is a sort of supplement to language affording a means of thought about form and quantity and a means of expression, more exact, compact, and ready than ordinary language. The great body of physical science, a great deal of the essential facts of financial science, and endless social and political problems are only accessible and only thinkable to those who have had a sound training in mathematics, analysis, and the time may not be very remote when it will be understood that for complete initiation as an efficient citizen of the new great complex world-wide states that are now developing, it is necessary to be able to compute, to think in averages and maxima and minima as it is now to be able to read and write." Those words were not written by a mathematician, but by a literary man of broad interests. It may be well for us to consider whether we should not, as teachers of mathematics, give our students such appreciation of the place of mathematics in education, as is implied in the statement of Mr. Wells.

Group A.

I. Review Topics in High-School Mathematics.

The review is to be made in such a way that the student is at

the same time taking a step forward. This includes:

1. Factoring, fractions, exponents, radicals, timed drills in arithmetic.
2. Plotting points and extension of number concept to include the negative. Rectangular coordinates only.
3. Linear equations.
    a. Solved by graphs, meaning of solution.
    b. Solution of two simultaneous equations, by determinants. Use of determinants in finding the area of a polygon.
4. Theorems from geometry.
    a. Right angled triangle, when one angle equals 30°, 45°, 60°.
    b. Theorems on parallel lines cut by a transversal.
    c. Theorems on congruency and similarity.
    d. Pythagorean theorem.
5. Quadratic equations in one unknown.
    a. Solution by graphs, with a discussion of the nature of the roots and meaning of a solution.
    b. Solution by the formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ with a discussion of the discriminant and the extension of the number concept to include the imaginary.
    c. Evaluation of formulas.
        (1) Falling bodies, projectiles, similar figures.

II. Trigonometric Functions of Any Angle.

1. Definitions. Sin A $= a/c$, Cos A $= b/c$.
2. Functions of angles 0°, 30°, 45°, 60°, 90°, 120°, 135°, etc.
3. Functions of $90°-x$, $180° \pm x$, $360°-x$.
4. Table of natural functions.
5. Abundance of applied problems, i.e., verbal problems.
6. Graphs of trigonometric functions.
    a. Use in electricity, wave motion, etc.

III. A Study of Straight Line Formulas in Analytics.

1. Distance between two points.
2. Mid-point of line.
3. Slope of a line, lines parallel, lines perpendicular.
4. Slope point form.
5. Slope intercept form.
6. Two point form.
7. Two intercept form.
8. Distance of a point from a line.

IV. Ratio, Proportion and Variation.

1. An abundance of applied problems in geometry, mechanics, physics, and in determining distances.

V. Trigonometric Functions of a Given Angle.

1. Fundamental relations.
   $\sin x = 1/\csc x$. $\cos x = 1/\sec x$, etc.
   $\sin^2 x+\cos^2 x = 1$. $1+\tan^2 x = \sec^2 x$. $1+\cot^2 x = \csc^2 x$.
   $\tan x = \sin x/\cos x$. $\cot x = \cos x/\sin x$.
2. Identities.
3. Application to surveying, physics, navigation, etc.

VI. Simultaneous Equations, One or Both of the Second Degree. No XY Term.

1. Method of drawing graphs.
   a. Use of points.
   b. Mechanical devices. Use of compass, right triangle, focal radii, asymptotes.
2. Solution by graphs with a discussion of the number of roots, condition of tangency, imaginary intersections.
3. Solution by substitution, or when possible by determinants.
4. Careful study of circle and parabola.
5. Applications to architecture, etc.

VII. Trigonometric Functions of $(X\pm Y)$. Sine, Cosine, Tangent.

1. Development of formulas.
2. Identities.
3. Applications, angle between two lines, etc.

VIII. Binomial Theorem.

1. Proof for positive integral exponents only.

IX. Logarithms.

1. Index laws including positive, negative, and fractional exponents.
2. Use of logarithms in computations with arithmetical numbers.
3. Use of logarithms in evaluating trigonometric expressions.
4. Tables. Logarithms of numbers, logarithms of trigonometric functions.

X. Solution of Oblique Triangles.

1. Law of sines.
2. Law of cosines.
3. Area of a triangle. Area $= \frac{1}{2}bc \sin A$
   Area $=\sqrt{s(s-a)(s-b)(s-c)}$.

The attention of teachers may be called to the fact that it is not absolutely necessary to include the law of tangents or the half angle formulas; in case of omission stress should be laid upon the law of cosines, and the solution of triangles by breaking them up into right triangles.

XI. Simple Introduction to Differential Calculus.

1. Rules for finding derivatives of a few simple functions.
2. Slope of lines and curves.
3. Distance, velocity, acceleration of falling bodies.
4. Maximum and minimum.
5. Rates.

This proposed brief and simple introduction of the elements of differential calculus into a freshman course is an experiment that is being tried with success in certain colleges of the state, but is still in the experimental stage so far as wide experience is concerned.

XII. History and Biography.

Appropriate historical notes should be scattered throughout the text. Recreation problems should not be omitted, especially those used to call attention to valuable mathematical principles.

1. History of trigonometry, its use today.
2. Discovery of logarithms.
3. Development of exponents.
4. Heron, Napier, Briggs, Newton, Descartes.

XIII. The above outline presupposes a rather free use of the principles of plane geometry and elementary mechanics, in the applied work. Geometric forms and surveying instruments may be used for illustrative purposes.

Group B.

I. Permutations and Combinations.

1. Applied problems.

II. Use of Formulas in Elementary Mechanics as a Better Basis for Mechanics, Physics, Science, Etc.

III. Theory of Equations.

1. Descartes rule of signs.
2. Horner's method.

IV. Further Study of Conics. *XY* Term Lacking.

1. Parabola and its applications.
2. Ellipse and its applications.
3. Hyperbola and its applications.

4. Discussion of 1, 2, and 3, with reference to bridges, buildings, eccentrics, orbits of planets, etc.

V. SIMPLE INTRODUCTION TO THE ELEMENTS OF INTEGRAL CALCULUS.
1. Areas.
2. Surfaces.
3. Volumes.
4. Inverse rate problems.

GROUP C.

I. PROBABILITIES.
1. Applications to life insurance, the Mendelian theory of inheritance, etc.

II. TANGENTS AND NORMALS.
1. Point form.
2. Slope form.

IIIa. ORTHOGRAPHIC PROJECTIONS.
1. Plotting of points, and lines first, second, third, and fourth angles with some application to mechanical drawing.

or IIIb. SOLID ANALYTIC GEOMETRY.
1. Equation of a plane.
2. Equation of a straight line.
3. Equation of a quadric surface when the axes are most conveniently located.

IV. HISTORY AND BIOGRAPHY. Suggested topics.
1. Work of early Greeks in developing conic sections.
2. Famous problems of antiquity.
3. Thales, Copernicus, Kepler, Archimedes, La Place, Euler, Vieta.

It is suggested that teachers will find the use of a limited amount of historical material of distinct value in the way of enriching the course.

E. E. Watson, Parsons College, Chairman; Ira S. Condit, Iowa State Teachers College; R. B. McClennon, Grinnell College, and H. L. Rietz, Iowa State University, Committee.

TIME, RATE AND DISTANCE PROBLEMS.

BY JOS. A. NYBERG,

Hyde Park High School, Chicago.

The object of the present paper is to call attention to a few rules and methods which have been of considerable help in teaching "time, rate, and distance" problems of elementary algebra. The textbooks show the relation between d, r, and t, and some algebras advise the use of a chart containing columns for d, r, t under which are to be entered the data for A and B, for the slow and fast trains, etc. The difficulty for the pupil, as in most prose problems, is in learning where to begin his attack, and how to derive the necessary equation from the data. In the method to be explained the chart is used, and four rules are given for its use, the underlying idea being that the six spaces in the chart must be filled *in a certain order.*

In the illustration below, the small numbers in the corners indicate the order in which the data was entered.

A and B travel toward each other from points separated by 250 miles, A at a rate which exceeds B's by 8 miles an hour. If they meet in 5 hours, at what rate did each travel?

	t	r	d
A	$5 \quad {}_{3}$	$r+8 \quad {}_{2}$	$5(r+8)$
B	$5 \quad {}_{4}$	$r \quad {}_{1}$	$5r$

In this chart the space corresponding to the unknown element is always the first to be entered. Or, as I state it to the pupils, the first rule is:

That quantity which is asked for in the problem must be entered first.

This rule is consistent with the pupil's previous experience for he always lets $x =$ the unknown quantity. But the letter x is avoided as it carries with it no connotation: we can forget whether it represents a distance, rate, or a time. If the problem asks: "What is B's rate?" enter r in the proper column and opposite B; if the problem says: "How far did A travel?" enter d opposite A, etc. If, as above, the question is: "What is the rate of *each*?" let r denote the rate of the slower one, although after the first weeks r may be either the smaller or greater rate.

The second rule is: *Enter next whatever data you are absolutely sure of, but do not make more than two entries on any one horizontal line.*

Thus, in the illustration above, the distance 250 can not appear in the chart as it is neither A's distance nor B's. Many problems will contain some such number, and the pupil learns to pass it by and look for some other entry. If the truth of his entry can not be questioned, then he need have no hesitancy about entering it; i.e., he need not hesitate and think "perhaps this is the number to be reserved for use in making the equation." Attention may also be called to the fact that entries usually are made by columns: the second entry is above or below the first, the fourth is in the same column as the third. Further, the latter part of the rule limiting the entries to two on a line is the more important thing to remember, as we shall see in applying the next rule:

The remaining column is always filled by using the relation $d = rt$ and never filled by using any information given in the problem.

Thus, when four entries have been made, the pupils can close the book until the remaining column has been filled. And it must be filled "from the head," as I say to the pupil, not "from the book." The teacher should vary the problems continually so that once the d column will be filled last by multiplying r and t, another time the t column found by dividing d by r, etc. No two successive problems should be the same. The pupil's work has been mostly mechanical up to this point in the problem. Here he has the opportunity to think, to exercise his reasoning abilities in deciding upon what quantities are to be multiplied or divided. Because of this opportunity and necessity, these problems are to be considered very desirable in helping to diminish the formal character of algebra.

When the six entries have been made (the first by the unknown letter, three more by the given information, the last two by means of $d = rt$) then we can always form the equation by the fourth rule:

The equation is found by relating the last two entries, together with any unused data in the problem.

In the illustration above, the number 250 has not been used; the equation is $5(r+8)+5r = 250$.

In all prose problems the equation is the most difficult part. One trouble is that from the various quantities or numbers

mentioned in the problem, the pupil must be taught to select those which form the equation. This difficulty has here been eliminated, for the pupil knows that the equation can involve only certain terms: the ones in his last column. For most prose problems no such simple rule has been found. It is a good idea to have the pupil draw heavy lines around his last two entries so that attention may be directed to them and detracted from the previous entries. The pupil who can complete the chart by these rules and then miss the equation, misses it because he overlooks some such obvious fact as that people must meet at the same time, or at the same place, or cover equal distances if they start and stop at the same places and have traveled the same road. But for the benefit of such weak pupils, it is possible to add to the four rules the following hint[1]: *If every number in the problem has been used, then most likely the last two entries should be equated; if there is a remaining used number, it is most likely equal to the sum or difference of the last two entries.* Only the weakest pupil should have his attention called to this hint for otherwise all the work will become too mechanical.

The advantages of the four rules can best be seen by comparing this method with the usual ones. Suppose the pupil writes:

$$\begin{aligned} \text{Let } r &= \text{B's rate} \\ \text{Then } r+8 &= \text{A's rate} \\ 5r &= \text{B's distance} \\ 5(r+8) &= \text{A's distance} \end{aligned}$$

Apparently this looks like a simpler method. The trouble is that the first two lines do not suggest the third. Blank paper will not stimulate thought, but an empty column for t will suggest that something should be entered in that space. Writing the first two lines in the solution above does not suggest the third and fourth lines as quickly as does the chart. Thus the main value of the chart is its suggestiveness.

A second noteworthy advantage is its adaptability to every type and variation of problem involving d, r and t. There are in general six kinds of problems: either d, r or t may be the unknown quantity, i.e., the first entry (see rule 1) may be in any one of three columns, and the other information (see rule 2) may then fall into either one of the two remaining columns. And to all these types the same method of solution can be applied.

[1]This suggestion applies to *all* problems found in the elementary texts. It is possible of course to devise problems for which it would not apply.

Even a more complicated problem leading to simultaneous quadratics can best be studied by the same rules. Consider for example the problem:

A starts from P to Q, traveling twice as fast as B, who is traveling from Q to P. After they meet, A reaches Q in two and one-half hours and B reaches P in ten hours. If the towns are three hundred miles apart, find the rate of each.

This problem requires the two following charts, the numbers in the corners showing the order of the entries.

Before meeting.

	t	r	d
A		$2r$ 2	$300-d$ 4
B		r 1	d 3

After meeting.

	t	r	d
A	$5/2$ 4	$2r$ 2	
B	10 3	r 1	

From the first chart $(300-d)/2r = d/r$.

From the second chart $5/2 \cdot 2r+10r = 300$.

If, in the second chart, the entries in the d column had been made third and fourth, and the t column had been entered last, the pupil would be led to the equations $d/2r = 5/2$ and $(300-d)/r = 10$.

Again, if in the first chart it had not occurred to the pupil to use d as one of the two unknowns, and he had entered the letter t for both A and B, his first chart would lead to the equation $2rt+rt = 300$ and his second chart to $5r+10r = 300$.

Consider next the old "army and messenger" problem: A messenger starting at the rear of an army reaches the front in a half hour. From there it takes him one-third of an hour to reach the rear. If he travels twenty miles per hour, how long was the army? The easiest solution involves calling r the rate of the messenger and R the rate of the army. Then $r-R$

represents the rate at which the messenger gains on the head of the army when moving forward, and $r+R$ is the rate at which he catches up to the rear on his return journey.

But this is the analysis of an expert. Let us see what the beginner can learn by his use of the charts. I explain to the class that the proposer of the problem could have asked just as well "At what rate was the army moving," or "Where was the van of the army when the messenger arrived at the rear?" instead of asking "How long was the army?" But in all events, any such question could be answered if we know the elements t, r, d, i.e., *every problem involves certain elements upon which all others depend, and knowing which, all other questions can be answered.* Here I like to sermonize a bit and explain that all problems of life should be analyzed in that way, and that even after leaving school, this attitude of analysis is the valuable element in our training; that in every situation of life we should inquire what are the fundamental elements which decide the destiny of the whole. Hence, let us prepare the charts, enter in it the elements we know and then decide on what we shall choose as our unknown element. The charts would appear thus:

Moving forward.

	t	r	d
Messenger	½	20	
Army	½		

Moving to the rear.

	t	r	d
Messenger	⅓	20	
Army	⅓		

As we need to have four spaces filled, *the charts suggest* that we enter the letter r for the rate of the army even though the problem does not specifically call for it. This entry will make two columns complete. The d columns can then be filled, and these columns are to be used in making the equations. We must remember that these entries under d would stand for the distance the army or messenger moved during the time entered in the t column. Now in each case the distance covered by the messenger depends upon the length (l) of the army. In the first chart we conclude $\frac{1}{2}\cdot 20=\frac{1}{2}\cdot r+l$. The second chart tells $\frac{1}{3}\cdot 20=l-\frac{1}{3}r$.

To summarize the points involved: first, we have four definite rules for teaching beginners how to attack the elementary problems; second, problems involving simultaneous equations yield to the same mode of attack; third, it is possible by these problems and charts to teach something of analysis, and to develop toward all problems a certain attitude of mind which will be useful after the algebra has been forgotten. This is the valuable part of the subject whose presence in a curriculum teachers are frequently called upon to justify.

SOME APPLICATIONS OF THE VARIOUS FORMS OF ZERO AND UNITY.

By Professor Richard Morris,
Rutgers College, New Brunswick, N. J.

We have the symbols 0 and 1 representing zero and unity respectively. In subtraction we also know that when a quantity is subtracted from its equal, the remainder is zero. This latter expression for zero, however, very often serves as a useful device in algebraic operations when the mere symbol would be of no service.

Unity may also have different forms of representation, such as x^0, $\sin^2 a+\cos^2 a$, $\log_e e$, $(-1)(-1)$, i^4, besides many others. When any quantity is divided by itself, the quotient is unity, whether the operation be performed or indicated. And this indicated form of unity is likewise a helpful device in effecting algebraic transformations. Since a quantity may be multiplied or divided by unity without altering its value, the multiplier or divisor may be written in any one of the various forms instead of by the symbol 1.

Some examples will serve to illustrate our meaning in the use of these indicated forms:

1. In solving $ax^2+bx+c = 0$ for x, we may use the form b^2-b^2 for zero. Multiplying the equation by $4a$, we get $4a^2x^2+4abx+4ac = 0$. Evidently if b^2 is added to the terms $4a^2x^2+4abx$ there is obtained the square of $(2ax+b)$. Hence, if b^2 is both added and subtracted, that is, if we write zero as b^2-b^2, we may add it at a convenient place in the equation without destroying the equation.

The writer appreciates that the ability to find a third term to combine with two given terms in order to form a trinomial which is a perfect square is involved in this problem. But the ability to complete the square is *very* essential and should be stressed.

2. In order to factor the expression $x^4+a^2x^2+a^4$, it will be necessary to write zero in the form $a^2x^2-a^2x^2$. This is evident since $2a^2x^2$ in place of a^2x^2, changes $x^4+a^2x^2+a^4$ into the square of x^2+a^2. We then have $(x^2+a^2)^2-a^2x^2$, which is easily factored.

3. Let it be required to change $x^4+2x^3-x^2-2x-3 = 0$ into the form of a quadratic. We may write it $(x^4+2x^3)-(x^2+2x)-3 = 0$. If now zero be written in the form x^2-x^2, the equation takes the form $(x^2+x)^2-2(x^2+x)-3 = 0$ which is evidently in the form of a quadratic, since one parenthesis is the square of the other. In order to change it still further and remain a quadratic, put $x^2+x = y$. It then becomes $y^2-2y-3 = 0$.

4. Similarly, change $2x^2-4x+3\sqrt{x^2-2x+6} = 15$ into the form of a quadratic. It is evident that the part $2x^2-4x$ must be built up so as to be identical with the radicand x^2-2x+6 save for a possible coefficient. Taking out the common factor 2 we get $2(x^2-2x)$. Zero in the form $6-6$ will effect the necessary transformation for the trinomial and will make a change in the constant term. Thus we will get $2(x^2-2x+6)+3\sqrt{x^2-2x+6}-12-15 = 0$, or $2y^2+3y-27 = 0$ by making $x^2-2x+6 = y^2$.

5. Take this example from analytical geometry, viz., $x^2+y^2-6x-10y+9 = 0$ which is the equation of a circle, the problem being to find the coördinates of its center and the length of the radius. By completing the square for those terms involving x and y separately, we see that zero must have the two forms $9-9$ and $25-25$, respectively. Using these we get $x^2-6x+9-9+y^2-10y+25-25+9 = 0$ or $(x-3)^2+(y-5)^2 = 25$. The form $9-9$ is not necessary if one observes that x^2-6x+9 is one of the two perfect squares needed.

6. If r is a root of $ax^2+bx+c = 0$, which means that ar^2+br+c is a form for zero, we can show that $x-r$ is a factor of $ax^2+bx+c = 0$.

Taking the identity $ax^2+bx+c = ax^2+bx+c$ and subtracting zero from the right member but in the special form above we get $ax^2+bx+c = ax^2+bx+c-ar^2-br-c$ or $ax^2+bx+c = a(x^2-r^2)+b(x-r)$.

7. Take this problem from the calculus, in which it is required to put $x^2+6x+13$ in the form of the sum of two quantities, the expression in terms of x being a perfect square. Zero must be written in the form $9-9$, in which case we get $x^2+6x+9-9+13$ or $(x+3)^2+4$.

8. Put $8+4x-4x^2$ in the form of the difference of two quantities, the subtractive one being a perfect square. In this case, zero must have the form $1-1$, when we get $9-(2x-1)^2$.

9. Put x^2-6x+5 in the form of the difference of two squares. Zero must be written $9-9$, thus giving $(x-3)^2-4$.

Thus we might multiply examples to show that zero in its various forms is a useful device and of frequent application. In our judgment, the value of this should be impressed upon the pupil and he should be encouraged to be on the alert for opportunities to employ it. Surely it will mean the exercise of his ingenuity and the satisfaction in using it will impart a relish for things mathematical.

There are many problems in which unity may be introduced, written as the quotient of two equal quantities, in order to effect transformations in solving or factoring. The following examples will illustrate:

10. In the expression $3x^2+7x+5$, take out the common factor 3. It is not apparent that there is such a common factor, but if unity is written in the form $3/3$ and applied to $7x$ and 5, we get $3x^2+3\cdot7x/3+5\cdot3/3$ or $3(x^2+7x/3+5/3)$.

11. Let it be required to put the final form of example 10 in the form of the difference of two squares in order to factor it. In this, zero must be written $49/36-49/36$. Applying this we get, $3[x^2+7x/3+49/36-49/36+12/12\cdot5/3]$, unity being written $12/12$. This condenses to $3[(x+7/6)^2+11/36]$. Again unity may take the form $(-1)(-1)$, and for one of these factors i^2 may be substituted. This gives $3[(x+7/6)^2-i^2 11/36]$ whose factors are $3[x+7/6+i\sqrt{11/6}]$ $[x+7/6-i\sqrt{11/6}]$.

12. Express ax^2+bx+c as the difference of two squares. Write unity as a/a and apply it to bx and c. Taking out the common factor a and writing zero as $b^2/4a^2-b^2/4a^2$, we get $a[x^2+bx/a+b^2/4a^2-b^2/4a^2+c/a]$ or $a[(x+b/2a)^2-\overline{b^2-4ac}/4a^2]$ whose factors are $a(x+b/2a-(\sqrt{b^2-4ac})/2a)$ $(x+b/2a+\sqrt{b^2-4ac}/2a)$.

13. Take the interesting problem to show that in the division of fractions, multiplication is substituted for division by inverting the terms of the divisor.

Take $\dfrac{a/b}{c/d}$ as the fraction. In this, unity must take the form

of $\dfrac{d/c}{d/c}$. When the pupil has reached division of fractions, he will have had practice in cancellation and will have had the definition of the reciprocal of a number, both of which ideas are involved in this problem. Using the above form of unity, we get

$\dfrac{a/b}{c/d} \times \dfrac{d/c}{d/c}$ or $a/b \times d/c$ by cancellation.

14. In order to change $\sqrt{7/5}$ into a mixed surd, unity must have the form $5/5$ or $7/7$. Thus we have $\sqrt{7 \cdot 5/5 \cdot 5}$ or $\sqrt{7 \cdot 7/5 \cdot 7}$ which become $1/5.\sqrt{35}$ or $7/\sqrt{35}$.

15. Rationalize the denominator of $(5+\sqrt{3})/(5-\sqrt{3})$. A knowledge of conjugate quantities is essential to see that unity must be written $(5+\sqrt{3})/(5+\sqrt{3}.)$

16. Transform the binomial $(8+12x)^{\frac{2}{3}}$ into a form with unity for the first term. In order to take out the common factor 8, unity must be written $2/2$, $4/4$, or $8/8$. Applying the last form we get $(8+8/8 \cdot 12x)^{\frac{2}{3}}$ or $4(1+3x/2)^{\frac{2}{3}}$.

17. Let it be required to express the coefficient of the middle term of $(x+y)^{28}$ in the form of factorial quantities. This coefficient is $28 \cdot 27 \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad .15/\underline{|14}$, whence it is evident that unity must be written $\underline{|14}/\underline{|14}$. Applying this to the coefficient we get $\underline{|28}/\underline{|14}\,\underline{|14}$.

18. Show that $\sqrt{(1-\cos x)/(1+\cos x)} = (1-\cos x)/\sin x$. If the pupil is ready with his trigonometric formulae, he will see at once that unity must have the form $(1-\cos x)/(1-\cos x)$, and when this is applied to the radicand, we get

$\sqrt{(1-\cos x)^2/(1-\cos^2 x)}$ or $(1-\cos x)/\sin^2 x$.

19. Change the following fractions to similar fractions, $a/(x+y), b/(x-y), c/(x^2-y^2)$. A knowledge of the lowest common denominator would suggest that unity in the first fraction must have the form $(x-y)/(x-y)$ and in the second fraction it must be $(x+y)/(x+y)$. In orderto change the sign of a fraction, unity must be written $-1/-1$.

20 As a final problem, let the coefficients of $ax^4+bx^3+cx^2+dx+e=0$ be integers. In order that this may be put in the form of a quadratic, we must have

$8a^2d = 4abc-b^3$.

Taking out the common factor a, we have

$a[x^4+(b/a)x^3+(c/a)x^2+(d/a)x+e/a]=0$, and building up

a trinomial perfect square, using $x^4+(b/a)x^3$ as part of the trinomial, we get
$a[x^4+(b/a)x^3+(b^2/4a^2)x^2-b^2x^2/4a^2+cx^2/a+(d/a)x+e/a]$
$= 0$. Writing unity in the forms, $4a/4a$ and
$(4ac-b^2)/4a^2 \cdot 4a^2/(4ac-b^2)$, and multiplying the terms cx^2/a and ax/a by these respectively, we get,
$a[x^4+(b/a)x^3+b^2x^2/4a^2+4acx^2/4a^2-b^2x^2/4a^2+$
$4ac-b^2/4a^2 \quad \cdot \quad 4a^2/4ac-b^2 \quad \cdot \quad (d/a)x+e/a] = 0$ or
$a[(x^2+bx/2a)^2+(4ac-b^2)/4a^2(x^2+4adx/4ac-b^2)+e/a] = 0.$

Hence this will take the quadratic form only when
$4ad/(4ac-b^2) = b/2a$ or if $8a^2d = 4abc-b^3$.

These problems suffice to exhibit the wide range to which the various forms of unity and zero may be applied, and the constant effort on the part of the pupil to discover opportunities for their application will mean much to him in his mathematical career.

ATOMIC THEORY IN BASEBALL TERMS.

By Dr. Irving Langmuir.

According to our present views, all forms of matter are built up of atoms, but we no longer regard these atoms as indivisible nor even as simple structures. If a lump of ordinary matter the size of a baseball could be magnified to the size of the earth, the atoms in it would then have become about the size of baseballs. In other words, an atom is about as big compared to a baseball as the baseball is when compared to the earth. The atoms are constructed of particles of positive and negative electricity arranged in a very open structure. All the positive electricity is concentrated into a very small particle, called the nucleus, located at the center of the atom. The negative electricity exists in the form of electrons which arrange themselves in space about the nucleus. The size of the electrons and nucleus is small compared with that of the atom itself. Thus if we imagine an atom magnified until it has a diameter of one mile, the electrons would be about five feet in diameter while the nucleus at the center would be only the size of a walnut.

The electrons in different kinds of atoms are alike but there are as many different kinds of nuclei as there are chemical elements, that is, about 92 in all. These differ from one another only in the amount of positive electricity they contain. Thus for the simplest element, hydrogen, the nucleus has a unit positive charge which is able to neutralize the charge of a single electron. A hydrogen atom then consists merely of the nucleus and a single electron. The next element, helium, has a nucleus with a double positive charge and the atom thus contains two electrons. In a similar way we find that the atoms of carbon have six electrons, while oxygen has eight, aluminum, thirteen, sulfur, sixteen, iron, twenty-six, copper, twenty-nine, silver, forty-seven, gold, seventy-nine, lead, eighty-two, and radium, eighty-eight electrons.

These electrons do not revolve around the nucleus the way the earth revolves around the sun, but they are arranged in three dimensions in a series of layers or concentric shells surrounding the nucleus. The electrons are probably not stationary but each revolves in its own orbit

about a certain equilibrium position. However, as we do not yet know much about these orbits we can speak of the positions of the electrons in the atoms as though the electrons were located in these equilibrium positions.

The first two electrons in any atom form the first shell about the nucleus, that is, two electrons are much closer to the nucleus than any of the others. In atoms with more electrons the next eight electrons form the second layer; then comes another layer of eight. If there are still more electrons these arrange themselves in a layer of eighteen followed by a second layer of eighteen and finally there may be an outside layer of thirty-two electrons. It is the successive formation of these various layers which causes the similar or recurring properties among the chemical elements which underlie the Periodic Table of the elements that is of such fundamental importance in chemistry.

The eight electrons in the second and third layers are arranged in a symmetrical way like the arrangement of the eight corners of a cube. This stable group of eight electrons is called the Octet. The chemical properties of the elements result from the tendency of the individual atoms to take up or give up electrons in order to form these octets. That is, the atoms strive to take certain stable configurations characterized by geometrical symmetry. They accomplish this in some cases by exchanging electrons with each other, while in other cases the atoms share pairs of electrons with each other—a sort of cooperative plan. The pairs of electrons thus constitute the chemical bonds between atoms which play such a prominent part in chemistry.

This theory of atomic structure and chemical combination not only explains an enormous number of chemical laws which have been obtained by experiment, but it leads to important extensions and in some cases to modifications of these laws, while in other cases it has led to new relationships and has made it possible to predict correctly the properties of certain substances before these properties have been determined by experiment.

TEACHERS SUFFER MOST.

Among those employes who suffer most acutely have been the teachers in our schools. Their situation in many parts of the country has become deplorable. Thousands of them, trained in their profession, with a high and honorable pride in it, have been literally forced to leave it, and to resign what had been their hope, not of wealth, but of loyal service in building the foundation of knowledge and character upon which our national strength must rest.

In consequence there is everywhere a shortage of teachers. An inquiry made by the Bureau of Education showed that in January, 1920, more than 18,000 teachers' positions in the public schools of the country were then vacant because the teachers to fill them could not be had. Over 42,000 positions are filled, in order that they may be filled at all, by teachers whose qualifications are below the minimum standard of requirement in the several states. It is the estimate of the Commissioner of Education that more than 300,000 of the 650,000 school teachers of the country are today "below any reasonable minimum standard of qualifications."

Many of those who remain in our schools receive less pay than common laborers, despite the long years of preparation for their profession that they have undertaken. This situation is a national menace. It is useless to talk of Americanization and of the diminution of illiteracy and other national educational problems, unless it is faced at once.—[Report of the Industrial Conference, called by the President.

CLASSIFICATION OF SCHOOL SUBJECTS BASED ON EDUCATIONAL FUNCTION AND VALUE.

By Lyman C. Wooster.

State Normal School, Emporia, Kansas.

Pearson, Thorndike and many others have investigated the influence of heredity in determining what we are mentally and physically, and have found that more than one-half of our mental and physical characteristics, probably three-fourths, come to us from our ancestors through heredity. The remaining one-fourth or more we get through home training, by contact with our environment and through education in the schools.

This fourth, though small, is very important, for it determines the sphere of our activities, equips us with a knowledge of the more important discoveries of the race and furnishes us with the latest and best methods of gathering and storing information. The schools have greatly increased their usefulness in recent years by teaching the boys and girls the scientific or inductive method of Bacon and by abandoning an overemphasis of the deductive method of the school men of the Dark Ages.

As teachers now give less exclusive attention to form and content and more attention to function and use it becomes necessary to shift the classification of subjects taught in the high schools to the later basis. The following classification, based on function and use, is offered by the writer for criticism and suggestion.

1. Mechanistic.—These subjects have for their chief purpose the teaching of habits of skill, either of mind or body, and of storing the memory with the useful wisdom of the race, all resulting eventually in conserving the energy of the body, including the brain, by making many of its activities automatic. The school subjects that naturally go here are spelling, typewriting, penmanship, the tables in arithmetic, traditional farming, all subjects taught so as to give skill without conscious thought in the various trades, most laboratory work, and botany, zoology and paleontology, taught chiefly for the ready identification of specimens.

2. Vocational.—These subjects prepare for those professions and occupations which always require skill and thought power for their successful prosecution. All the biological and physical sciences can be taught so as to go here. The new sciences such as agriculture, home economics, commerce and manual training are now taught for thought as well as skill. Constructive and

informational English and some portions of arithmetic are fundamental in all the professions and occupations.

3. Avocational.—These subjects are intended to give breadth of knowledge and prepare a better citizenship. All school subjects have a more or less socializing effect, but the subjects which follow are specially adapted to this field: History, civics (well taught), political economy, sociology, English literature and all the sciences studied chiefly for information are avocational.

4. Cultural.—This much abused term was originally made to include merely the social graces. It meant music, painting, poetry, dancing, rhetoric, oratory, and grace of bearing and ease of conversation in social gatherings for people of wealth. People of leisure sought in all fields for material with which to entertain. Those who travel must know several of the foreign languages; hence French, German and Spanish were studied for convenience and as an accomplishment. The term cultural is used here in its original signification.

5. Specialistic.—Many years ago it was believed that the mind could be educated in high schools and colleges by causing it to master any subject, the less interesting and useful the better, and then that the new powers could be used in the easy conquest of the remaining subjects of the curriculum. So thoroughly convinced were the teachers of Latin and Greek of the truth of this theory that some of them inscribed above their classroom doors this warning, "Let nothing useful enter here." (—D'Ooge.)

The subjects that may be listed in this fifth group are—pure science, pure mathematics, Latin and Greek, and the deductive type of philosophy. It is now generally admitted by educators, that science for its own sake, higher mathematics, Latin and Greek literatures and deductive philosophy can be studied with profit only by specialists in universities. These experts in science, mathematics, the classics and in philosophy are mature in their mental powers and are capable of abstracting themselves from the world in which they live and confining their interests to a limited world in which they are the only inhabitants. This was illustrated by the German who spent a lifetime studying the Greek verb, and regretted when death came that he had not confined his studies to the dative case of the noun.

The specialist in science dares all, spends all, gives up all for his beloved botany, zoology, geology, physics or chemistry. The mathematician easily drifts into astronomy where he can

spend months and even years on a single problem. The philosopher is supremely happy when he can mount the cumulus clouds of his imagination and drift about in the realms of the absolute.

It is evident that classifications based on functions are not so rigid as those based on structure. People possessing the same general anatomy may function in widely different spheres of activity. In the above classification, botany and zoology, for example, may be taught as mechanistic subjects, when emphasis is placed on skill in the identification of plants and animals. Indeed, this was the only purpose in studying botany and zoology till within the past few years. Now, function and use are emphasized, as well as skill, and these subjects become vocational. Or, they may be so taught as to widen one's knowledge of his environment and to enable the pupil to readily indentify the important plants and animals as he sees them, and these subjects become avocational.

Cultured civilized man has always admired the beauty of flowers, and no social occasion is complete without floral decorations, and every yard must contain flowering plants. The painter has found in animal life abundant material for his canvas and the poet has sung some of his most delightful songs in praise of birds and insects.

This that is true of botany and zoology is more or less true of the other school subjects. They are not well taught, according to the modern educator, unless they serve to prepare the boys and girls for personal, social and civic usefulness and to enjoy all that is true and beautiful in their environments. Subjects which do not thus help to round out the lives of our young people should be dropped from our courses of study.

As to the specialistic group, the old idea that subjects should be mastered for discipline, even though they do not contribute to social efficiency, is productive of much misspent time in the schools. It has been demonstrated by Thorndike and many others that training acquired in mastering any subject can not be transferred and used in the mastery of another and different subject. Thus training acquired in mastering Latin and Greek can not be used in mastering algebra and geometry and the reverse. A small transfer of training is possible, however, when two subjects have identical elements. The identical element may be one of substance, as in mathematics and physics; it may be one of process, as in laboratory work in chemistry and zoology;

or it may be one of aim, as in all written work when the teacher makes neatness a quality of prime importance in all examination and notebook work. This last is especially effective when various emotions are aroused by a promise of rewards and punishments. In all these transfers the pupil must be made conscious that the element is identical in the two subjects or there is no transfer.

Akin to this practice of making boys and girls take traditional subjects chiefly for discipline is the one which requires the student to take pure science, pure mathematics and the classics in their entirety and for their own sake when he has no direct use for more than a very small portion of these subjects. The needs of the student are certainly paramount to the needs of the subjects, and must be observed more fully in constructing courses of study or there will be open rebellion on the part of our young people. In saying this, there is no thought of excluding specialistic subjects from the course of study of any college or university student who is making a general preparation to meet the diverse conditions of his environment. The highest success in any field of human activity is possible only when the student makes such preparation; but the educator says that that must be his object, with plenty of time in which to execute it.

TEST FOR COLOR BLINDNESS.

By Maud S. Stuart,

Faribault, Minn.

I have read in the March number of School Science and Mathematics that Mr. Turton of the Bowen High School, Chicago, gave a report at the association, on "Testing a Class for Color Blindness." For the last four years I have been paying special attention to training my pupils in physics and chemistry and paying more attention to the distinction of colors. We have found only one student who seemed to be color blind. I use the spectrum colored papers, which I purchase from Thomas Charles & Co., Chicago, buying the large sheets and each student makes his own charts, mounting 2×1 in. pieces of each of the eighteen colors—one of normal strength with two tints and two shades of each. I have a much larger chart on the wall, which I have made for reference. Then my pupils gather colored pictures, and mount them, with small pieces of paper, also mounted, showing the colors of inks used in printing the pictures. We do some work too, in finding the names of colors of samples of cloth. This year, I have purchased the Standard Color Chart of America from the Textile Color Card Association of U. S., New York City, a simplified list of about 120 colored ribbons, with names, and numbers, indicating the composition of the colors and the depth of color.

I think the majority of our young people are not color blind but they have not been trained thoroughly enough in noticing the various colors about them.

PROBLEM DEPARTMENT.

Conducted by J. A. Nyberg,

Hyde Park High School, Chicago.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and solve problems here proposed. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The Editor of the department desires to serve its readers by making it interesting and helpful to them. If you have any suggestion to make, mail it to him. Address all communications to J. A. Nyberg, 1044 E. Marquette Road, Chicago.

SOLUTION OF PROBLEMS.

646. *Proposed by A. Pelletier, Montreal, Can.*

Find the remainder of the division of 6^{592} by 11.

I. *Solution by N. Barotz, New York City.*

Investigation seems to show that when 6^{10n}, 6^{10n+1}, 6^{10n+2}, · · · · are divided by 11, the remainders will be 1, 6, 3, 7, 9, 10, 5, 8, 4, 2 and will repeat in this order. Hence $6^{592}/11$ gives a remainder 3.

II. *Solution by M. G. Schucker, Pittsburgh, Pa.*

$6^{592} = 36^{296} = (33+3)^{296} = 11N_1+3^{296}$

$3^{296} = 81^{74} = (77+4)^{74} = 11N_2+4^{74}$

$4^{74} = 16^{37} = (11+5)^{37} = 11N_3+5^{37}$

$5^{37} = 5 \cdot 5^{36} = 5(22+3)^{18} = 11N_4+5 \cdot 3^{18}$

$5 \cdot 3^{18} = 5 \cdot 9 \cdot 81^4 = 11N_5+5 \cdot 9 \cdot 4^4 = 11N_5+11 \cdot 1047+3$

III. *Solution by the Proposer.*

By Fermat's Theorem, 6^{10} = (multiple of 11) + 1

Then $6^{592} = (6^{10})^{59} \times 6^2$ = (multiple of 11+1) × 36

= (multiple of 11+1)(multiple of 11+3)

= (multiple of 11) + 3.

Also solved by Eugene M. Berry, Iowa City, Iowa, Harold G. Donovan, senior at Concord High School, N. H., Jas. H. Packham, Collegiate Institute, Owen Sound, Michael Goldberg, student at South Philadelphia High School, Pa.

647. *Proposed by Walter R. Warne, State College, Pa.*

$S_1, S_2, S_3, \cdots S_p$ are the sums of p arithmetical progressions each continued to n terms, the first terms are 1, 2, 3, · · · · p, respectively, and the common differences 1, 3, 5, · · · $2p-1$. Prove that $S_1+S_2+S_3+ \cdots +S_p = np(np+1)/2$.

I. *Solution by Harold G. Donovan, senior at Concord High School, N. H.*

The sum of the first series in $(n^2+n)/2$, of the second series is $1/3(3n^2+n)$, of the pth is $[(2p-1)n^2+n]/2$. Regarding these sums as the terms of a series for which the difference is n^2,

$$S_1+S_2+ \cdots S_p = p[1/2(n^2+n)+1/2(2p-1)n^2+1/2n]/2$$
$$= np(np+1)/2.$$

II. *Solution by Nelson L. Roray, Metuchen, N. J.*

$S_1 = 1+2+ \cdots = [2+(n-1)]n/2$

$S_3 = 3+8+ \cdots = [6+5(n-1)]n/2$

$S_p = p+(2p-1)+ \cdots = [2p+(n-1)(2p-1)]n/2$

$S_1+S_2+ \cdots S_p = [2+4+ \cdots 2p+$
$(n-1)(1+3+5 \cdots 2p-1)]n/2$

$= [(2+2p)p/2+(n-1)(1+2p-1)p/2]n/2 = np(np+1)/2.$

Also solved by N. Barotz, A. Pelletier, R. T. McGregor, Elk Grove, Calif., Jas. H. Packham, Michael Goldberg, and the Proposer.

648. *Proposed by J. W. Lyle, Hartwell High School, Cincinnati, O.*

In the right dihedral angle formed by a floor and a wall is built a rectangular coal bin four feet high which extends three feet from the wall. What are the possible positions of a ladder eleven feet long that will touch the wall, the bin and the floor at the same time?

Solution by Walter R. Warne.

Let x = distance from A, the foot of the ladder, to the front foot of the

bin, and y = distance on wall from the back top of the bin, B, to the top of the ladder. Then AB = 11, $xy = 12$, $(x+3)^2+(y+4)^2 = 121$, or $x^4+6x^3-96x^2+96x+144 = 0$. By Sturm's Theorem, there are only two positive real roots, and consequently only two positions of the ladder. Horner's Method of Approximation gives $x_1 = 2.2008_1+$, $x_2 = 6.99 \cdots$ Or, by Descartes' Rule of Signs there are not more than two positive real roots, and not more than two negative real roots. Also, $f(2) = +$, $f(3) = -$, $f(6) = -$, $f(7) = +$.

649. *Proposed by Walter R. Warne.*

If the inscribed circle of the triangle ABC passes through the center of the circumscribed circle, then

$$\cos A+\cos B+\cos C = \sqrt{2}.$$

I. *Solution by Jas. H. Packham.*

$\cos A+\cos B+\cos C = 1+4\sin A/2 \sin B/2 \sin C/2 = 1+r/R$. If I is the in-center and S the circum-center, $IS^2 = R^2-2Rr$. But $IS = r$. Hence $r^2 = R^2-2Rr$, or $2r^2 = (R-r)^2$, or $\sqrt{2} = R/r-1$. Then $R/r = 1+\sqrt{2}$, $r/R = \sqrt{2}-1$; $r/R+1 = \sqrt{2}$. Hence $\cos A+\cos B+\cos C = \sqrt{2}$.

II. *Comment by the Proposer.*

Obviously, the proposition is true for isosceles right triangles, and untrue for a $60°-30°$ right triangle or an equilateral triangle. The natural query would be: Is the theorem true for isosceles non-right triangles?

III. *Comment by the Editor.*

To answer the above query, put $A = B$, $C = 180-2A$, then $\cos A+\cos A+\cos 180-A) = \sqrt{2}$ or $2\cos^2 A-2\cos A+\sqrt{2}-1 = 0$; then $\cos A = \sqrt{2}/2$ or $(\sqrt{2}-1)/\sqrt{2}$. The first solution makes $A = 45°$, and the second solution will satisfy only if $\cos A+\cos B-\cos C = \sqrt{2}$. Hence, if the triangle is isosceles, it must also be a right triangle, and the hypotenuse is then a diameter of the circumscribed circle. The problem should now be put: given a circle passing through the center of a larger circle, and entirely inside the larger circle. Construct triangles for which the two circles will be the inscribed and circumscribed circles.

Also solved by A. Pelletier, N. L. Roray, and the Proposer.

650. *Proposed by Norman Anning, Orono, Me.*

Two hunters start from the same camp. A goes one mile east and then one mile N 18°E. B goes one mile N 30°E. and then one mile N 54°E. Find the distance and direction of B from A.

Solution by N. Barotz, New York City.

A goes east $1+\sin 18° = 1.30902$ mi.

B goes east $\sin 30°+\sin 54° = 1.30902$ mi.

A goes north $\cos 18° = .95106$ mi.

B goes north $\cos 30°+\cos 54° = 1.45382$ mi.

Hence B will be .50276 mi. directly north of A.

Comment by the Editor.

The solution above suggests that $\sin 30°+\sin 54° = \sin 18°+\sin 90°$, which ought to be derivable by trigonometry without tables.

Also solved by H. G. Donovan, R. T. McGregor, A. Pelletier, M. Goldberg.

LATE SOLUTIONS.

638. W. R. Warne, M. G. Schucker, Pittsburgh, Pa.
641. M. Goldberg.
643. N. Barotz, M. G. Schucker.
644. N. Barotz, M. Goldberg, M. G. Schucker.
645. M. G. Schucker.

PROBLEMS FOR SOLUTION.

661. *Proposed by Jas. H. Packham, Collegiate Institute, Owen Sound.*

Given an $\angle ABC$, a point Q on BC, and a point P between the sides of the angle. Construct an isosceles triangle, having its vertex on AB, its base on BC, one extremity of the base at Q, and the side opposite Q passing through P.

662. *Proposed by J. L. Riley, Stephenville, Texas.*

If a, b, c, d are the sides, and $\triangle$ the area, of a quadrilateral inscribed in a circle of radius R, prove that

$$16R^2\triangle^2 = (ab+cd)(bc+da)(ca+db).$$

663. *Proposed by A. Pelletier, Montreal, Canada.*

Prove that the product of four consecutive numbers can not be a perfect square.

664. *Proposed by N. P. Pandya, Amreli, Kathiawad, India.*

Three equal chords (length l) of a circle form, when produced both ways, a triangle ABC. Express the sides of the triangle in terms of its angles and l. Hence find under what circumstances the sides would be in geometrical progression.

665. *Proposed by W. R. Warne, State College, Pa.*

Solve $\cos(ax)\cdot\cos(bx) = \cos(a+c)x\cdot\cos(b+c)x$.

SCIENCE QUESTIONS.
Conducted by Franklin T. Jones.

The Warner & Swasey Company, Cleveland, Ohio.

Readers are invited to propose questions for solution—scientific or pedagogical—and to answer questions proposed by others or by themselves. Kindly address all communications to Franklin T. Jones, 10109 Wilbur Ave., Cleveland, Ohio.

ACKNOWLEDGMENTS.

Contributions not elsewhere mentioned have been received from:

Manual Training and Model High School, Fargo, N. Dak. (The contributor did not give his name.)

A. Haven Smith and class, Riverside Junior College, Riverside, Calif.

C. Harvey Waite, Gary, Ind.

Robt. W. Boreman, Georgia School of Technology, Atlanta, Georgia.

Rolland E. Klug, Bourbon, Ind.

QUESTIONS AND PROBLEMS FOR SOLUTION.

Please send in solutions of problems numbered 340, 341, 342, 343, 344, 345 in the list of questions that follows.

In the booklet from which these questions were taken there was a total of eighteen examination papers in British History, English, French, Italian, German, Latin, Greek, and Mathematics—no science whatever except this single examination in dynamics. How would our students make out with an examination of this sort? Please note, however, that success in such an examination can only be possible as result of *thorough instruction on a limited field.*

SCOTTISH UNIVERSITIES ENTRANCE BOARD PRELIMINARY EXAMINATIONS.
Dynamics.

Wednesday, 17th September, 1919, 3 to 6 p. m.

A Candidate may obtain FULL *marks by doing about two-thirds of this paper.*

Marks will not be awarded to answers where the work is not shown.

N. B.—*Candidates must write on their books the number of the table at which they sit.*

1. What is the difference between uniform velocity and average velocity? Find the angle between two equal component velocities if their resultant is equal to either of them. Compare the velocities of the extremities of the hour, minute, and second hands of a watch, their lengths being 0.6, 0.81, and 0.32 inches respectively.

2. A body, starting with a velocity (V) of 20 ft. per sec., moves for 6 secs. with an acceleration (ϵ) of 5 ft. per sec. per sec. Draw a graph showing relation between velocity of body at any instant and the time;

and prove that the area of the figure contained by the graph, the axes of coordinates, and any ordinate represents the distance travelled in a certain time. From your figure deduce the formula

$$s = Vt + \tfrac{1}{2}at^2.$$

3. State, without proof, how to find, both as regards magnitude and direction, the resultant of (i) two non-parallel coplanar forces, (ii) two parallel unequal forces acting in the same direction, (iii) two parallel unequal forces acting in opposite directions.

340. A uniform beam ABC, 24 feet long, which weighs 2 cwt. and supports two loads, each weighing 1 cwt., rests in a horizontal position on two posts, one at the centre B of the beam and the other at a distance of 10 feet from A. If one of the loads is 2 feet from A, find the distance from A of the other load when the thrust on the post at B is zero.

4. Explain, with two illustrative examples, what is meant by saying that a body has momentum. Find the ratio of the momenta of two bodies A and B in the following case: The mass of A is 1 cwt., and it moves at the rate of 30 miles per hour; the mass of B is 21 oz., and it moves at the rate of 2816 ft. per sec.

5. What is the centre of gravity of a body? Show that, if a body be freely suspended from a point O, its centre of gravity will be in the vertical line through O in the position of rest.

341. A uniform one-foot rod is broken into two parts of 5 and 7 inches, which are then placed so as to form the letter **T**, the longer portion being vertical; find the position of its centre of gravity.

6. Explain what is meant by (i) Work, (ii) Energy, (iii) Power; and state the units in terms of which each is measured, when the pound, the foot and the second are the units of force, length and time respectively.

342. A square shaft, of side 6 feet, is to be sunk to a depth of 60 feet in a uniform material of specific gravity 2.5. How long will a man take to do this, working with a windlass, if he does 70,000 ft.-lbs. of work daily, supposing the windlass to work without friction? [A cubic foot of water weighs 1000 oz.]

7. Define the terms: Kinetic friction, limiting friction, and coefficient of friction. Two inclined planes of equal length, the one rough and the other smooth, are tilted at angles θ and ϕ respectively, so that a body takes the same time in sliding down either. Find the coefficient of friction of the rough plane.

8. Enunciate Newton's Third Law of Motion, and give an illustration of its principle.

343. A pile, weighing half a ton, is driven $\frac{1}{4}$ inch into the ground by the blow of a hammer weighing 2 tons descending from the height of 4 feet. Find (i) the velocity of the hammer just before striking the pile, (ii) the velocity of the pile and hammer just after the blow, (iii) the average resistance of the ground.

9. Explain what is meant by "the centre of pressure" of a plane area immersed in water. If the area is a rectangle with one side in the surface of the water, where is the centre of pressure?

344. A tank with a rectangular bottom and vertical walls stands on a horizontal floor and is divided into two compartments by a vertical partition at right angles to its length. The width of the tank is 12 feet and the depth of the water in one compartment is 9 feet and in the other 6 feet. Find the magnitude and the line of action of the resultant thrust on the partition.

10. Two exactly similar straight glass tubes are fitted up as mercury barometers with their unsealed ends each resting in a bowl of mercury. Into the one is introduced a small quantity of air, and into the other a small iron ball whose diameter is less than the bore of the tube. Describe exactly and explain what will happen in each tube. If the pressure, volume, and Centigrade temperature of a gas change from a, b, c to d, e, f respectively, form an equation showing how these various quantities are related.

11. What is the Principle of Archimedes? Describe how it is applied to determine the specific gravity of a liquid.

345. A cylindrical diving-bell, 7 feet in height and 4 feet in diameter, is sunk in water till its top is 80 feet below the surface; if the height of the water barometer is 34 feet find the height to which the water has risen in the bell.

WHY not try the Panama Canal problem with your best class? The results will be interesting.

SOLUTIONS AND ANSWERS.

317. *Proposed by J. C. Packard, Brookline, Mass.*

An automobile, weighing with load 1,000 lbs., while running at the rate of 5 miles an hour collides with a telegraph pole. The fender is crushed in about 2 inches. How heavy a blow did the auto deliver?

[Query: Is this the piledriver problem in another form?—EDITOR.]

Solution by K. L. Pohlman, Cleveland, Ohio.

Before the machine is brought to rest it will have performed $1000\times 7.3^2/2\times 32.2 = 827.3$ ft. lbs. of work and as this energy is expended in a distance of 1-6 ft., the average force with which the machine strikes the pole equals, $827.3 \div \frac{1}{6} = 4963.8$ lb.

323. *Proposed by J. C. Packard, Brookline, Mass.*

What is the horse power required to drive an automobile weighing 2,000 lbs. up an 8% grade at the rate of 30 miles per hour?

Solution by H. W. Corzine, Cleveland, Ohio.

$$\text{Horse power} = \frac{\text{Ft. lbs. per minute moved vertically}}{33{,}000}$$

$$= \frac{2000\times(30\times 5280/60)\sin A}{33000} \quad \text{but } A = \tan^{-1}\frac{8}{100} = 4^\circ\text{—}35'$$

$$= \frac{2000\times 2640\times .07991}{33000}$$

$$= 12.78 \text{ Ans.}$$

REPORT OF THE WINTER MEETING.

The regular semiannual meeting of the Southern California Science and Mathematics Association was held in the auditorium of the Central Intermediate School, Los Angeles, on Thursday, December 18, 1919, at 1:30 p. m., President E. E. Chandler presiding.

The minutes of the previous meeting held at Venice, May 17, 1919, were read and approved. The treasurer's report was read, approved by the nominating committee, and accepted by the association.

Mr. Fisk, chairman of the nominating committee, explaining that the present officers had been elected at the last meeting and thus had served but a half year, recommended that they be reelected, and made his report as follows: For president, Mr. E. E. Chandler, of Occidental College; for vice president, Miss Olive Kelso, of Pasadena High School; and for secretary-treasurer, Mr. N. D. Knupp, of Santa Monica High School. The report was approved and the secretary instructed to cast the unanimous ballot of the association for the officers named. They were declared elected by the chairman.

Under the head of new business, Mr. Fluckey, of Lincoln High School, Los Angeles, presented the following resolution:

"Whereas the Great War has emphasized the need of simplifying and standardizing computation and measurement, and

"Whereas the sharpening of trade competition in Europe and in South America makes imperative the use of measurements with which these foreign purchasers are familiar, and

"Whereas it is well-known that the system of weights and measures in common use in this country is cumbersome, difficult to learn, and wasteful of time and energy, and

"Whereas the metric system by act of Congress was made the standard for this nation many years ago, but has gained its way but slowly against the prejudice of those unfamiliar with it, be it

"Resolved, That we, the Southern California Science and Mathematics Association, do hereby urge the immediate passage of bills by our state and national legislatures which will hasten the general adoption of this system;

"That we propose as one means to secure this desired end that all construction, bids and supplies of public property after July 1, 1922, be required by law to be in metric units;

"That copies of these resolutions be sent to our state and national representatives, and to the other Science and Mathematics Associations of the United States, and

"That we hereby pledge ourselves as an association and as individuals to the further prosecution of this cause."

Upon motion duly made and seconded the resolution was adopted.

The lecturer of the day, Professor Joel H. Hilderbrand, was then introduced and gave a very interesting and valuable talk on the subject "Some Phases of the Teaching of Chemistry." He discussed first the spirit and characteristics of science in general, and then explained ways of securing and holding interest, and methods of teaching the subject of chemistry.

There being no further business, the general meeting adjourned to reassemble by sections for the election of officers and section programs which were carried out according to the printed program attached.

N. D. Knupp, *Secretary.*

BOOKS RECEIVED.

The Carnegie Foundation for the Advancement of Teaching, Fourteenth Annual Report of President and Treasurer. 148 pages. 19×25.5 cm. Paper. 1919. 576 Fifth Ave., New York City.

Everyday Chemistry, by Alfred Vivian, Ohio State University. 560 pages. 13×19 cm. Cloth. 1920. American Book Company, Chicago.

Physiography, by Rollin D. Salisbury, University of Chicago. Third Edition. Pages xv+676. 14.5×21.5 cm. Cloth. 1920. Henry Holt and Company, New York City.

Armenia and the Armenians, from the earliest times until the Great War (1914), by Kevork Aslan, translated from the French by Pierre Crabiles. Pages xxix+138, 13.5×19.5 cm. Cloth. 1920. The Macmillan Company, New York City.

Zoology, text book, for colleges and universities, by T. D. A. Cockerell, University of Colorado. Pages XII+558. 14.5×21 cm. Cloth. 1920. $3. World Publishing Co., Yonkers-on-the-Hudson, New York.

Applied Arithmetic, Books II and III, by N. J. Lennis, University of Montana, and Frances Jenkins, University of Cincinnati. Pages IX+294 and IX+340. 13×19.5 cm. Cloth. 1920. Lippincott Publishing Co., Philadelphia.

Household Arithmetic, by Katherine F. Ball, University of Minnesota, and Miriam E. West, Girls' Vocational High School, Indianapolis. Pages 271. 14×20.5 cm. Cloth. 1920. P. B. Lippincott Publishing Co., Philadelphia.

From the Department of the Interior, Bureau of Education, Washington, D. C., the following bulletins:

Number 70, Schools and Classes for Feeble Minded and Sub-Normal Children.

Number 3, Private, High Schools and Academies.

Number 45, A Credited Report of Schools of North Central Association.

Number 77, Americanization.

Number 62, Extension work.

Number 80, Teaching English to the Foreigners.

Number 6, The Child and the Kindergarten.

BOOK REVIEWS.

Analytic Geometry, by Maria M. Roberts, Professor of Mathematics in Iowa State College, and Julia T. Colpitts, Associate Professor of Mathematics in Iowa State College. Pages x+245. 13×19 cm. 1918. John Wiley & Sons, Inc., New York.

The material of the usual course in analytic geometry is presented in this book with emphasis placed on those portions in which experience has shown the student of calculus to be most frequently deficient. Polar coordinates receive more than the usual attention, and transcendental and parametric equations receive due attention. The large number of exercises gives an opportunity of considerable choice on the part of the instructor. The well-drawn diagrams will prove of much use to the student.

H. E. C.

Fundamentals of High School Mathematics, by Harold O. Rugg, Columbia University, and John R. Clark, Department of Mathematics, Chicago Normal College. Pages xv+368. 14×19 cm. Price, $1.60. 1919. World Book Company, Yonkers-on-Hudson, N. Y.

The preparation of very few textbooks in mathematics has been based on such long continued and scientific study of the way in which children learn and the kind of mathematics that will prove of social worth to them as the authors of this book have made. As they have come to the conclusion that children do their thinking in detailed word-symbols this book is planned to make the transition from thinking in detailed word symbols to that in algebraic symbols so gradually as to keep, step by step, just ahead of the pupils' mental advance. The exposition of the text develops so gradually that the average pupil can read it and work the problems with little aid from the teacher.

The subject matter selected on two principles—social worth and thinking value—includes the use of the letters to represent numbers; simple equations; construction and evaluation of formulas; the finding of unknown distances by means of (1) scale drawing, (2) the principles of similar triangles, (3) the use of the properties of the right triangle (including sine, cosine, tangent); and the preparation and use of statistical tables and graphs. The selection of material for this course is quite in agreement with the recommendations of the National Committee on Mathematical Requirements.

Among the outstanding features of this book are: the careful explanations and development of new processes; the wise omission of formal material; the excellent presentation of word-problems; the unique organization of special products and factoring; the timed practice-exercises for developing skill in essential processes; and the emphasis upon the notion of relationship between variable quantities. Every teacher or school administrator who is interested in improving the educational value of the first-year high school mathematics course should examine this textbook.

H. E. C.

Everyday Chemistry, by Alfred Vivian, Dean of the College of Agriculture of the Ohio State University. Pages, 560. 13x19 cm. 1920. American Book Co.

Progressive educators are generally agreed that the studies of the high school should be organized in terms of modern life as found in the pupil's daily activities rather than organized as a logically arranged subject, particularly if the latter arrangement does not give the pupil a real insight into the purposes and values of the subject.

This viewpoint is not only desirable but necessary in order to get modern high school pupils to rub up against the educational grindstone. It is more than motivation. It is learning naturally with the need of knowledge being actually felt before the information is given.

Chemistry teachers will welcome Vivian's Chemistry since it turns the instruction of the subject in the direction of the above viewpoint. It omits some of the "classics" of chemistry to make room for the more practical. This is particularly true for the first half of the book, the treatment of metals and their compounds being excised most. This latter point is just as well for the cyclopedic listing of compounds of metals in the average text of the past was almost valueless. The atomic theory has only incidental treatment. It would be more appreciated by pupils if the main assumptions were deduced from facts rather than given in a dogmatic way. If the atomic theory is given any space it should have enough to make the mechanics of equation writing a better understood process. The author seems to use H or H_2 for hydrogen gas at will without reference to the facts of the case.

The last three-fifths of the book has a direct bearing on the chemistry of organic substances, a much larger treatment than is usually accorded this division of the subject. This is due to the author's desire to make the book useful for those schools that wish to take advantage of the provisions of the Smith-Hughes enactment for vocational instruction in Domestic Science and Agriculture. In this respect the book will give better satisfaction than any other.

The laboratory directions are to be found in the exercises at the close

of each chapter, and they are interspersed with questions to test the pupil's mastery of the information given in the text part. Some such method is necessary to direct the pupil's mental effort. This plan should succeed admirably if the pupil is held to account for his effort by satisfactory answers to the questions. It is another question regarding how much training the pupil will get from reading experimental data, when he can read so much easier from the text by turning back a few pages. The full value of individual experimental work can not be realized if a text is at hand to do the thinking and reasoning for the pupil. For this reason the laboratory directions must needs be separate from the text. Unless full value is realized the time and expense of individual laboratory work can not be justified. Vivian's Chemistry will contribute most to the advancement of chemical knowledge by helping teachers and pupils see the chemistry of life rather than the chemistry of books. H. R. S.

SOME INTERESTING FACTS CONCERNING THE RELATIONS BETWEEN THE THREE SIDES OF A RIGHT TRIANGLE.

By Robert R. Knowles,

Industrial Arts High School, Sterling, Colo.

In the past, the writer has often wished for some method by which problems in the solution of the sides of a right triangle by the Pythagorean Theorem could be assigned to different members of a class without the instructor having to waste time in checking the problems by working out each individual case. He has therefore hit upon the following plan which is original as far as he knows.

If we assume the identity,

$$k^4+2k^2+1 = k^4+2k^2+1,$$

by arranging and collecting we have

$$k^4-2k^2+1+4k^2 = k^4+2k^2+1,$$
$$(k^2-1)^2+(2k)^2 = (k^2+1)^2.$$

If then we take a triangle with k^2-1, $2k$, and k^2+1 as sides, we have by the Pythagorean Theorem, a right triangle because the sum of the squares of two of the sides equals the square of the remaining side. This may be carried out with a number of identities as shown in the table below.

Identity.	Sides. *a*	*b*	*c*
$k^4+2k^2+1 = k^4+2k^2+1$	(k^2-1)	$(2k)$	(k^2+1)
$k^4+8k^2+16 = k^4+8k^2+16$	(k^2-4)	$(4k)$	(k^2+4)
$k^4+18k^2+81 = k^4+18k^2+81$	(k^2-9)	$6k$	(k^2+9)
$k^4+32k^2+256 = k^4+32k^2+256$	(k^2-16)	$8k$	(k^2+16)

These sides may be written

a	*b*	*c*
k^2-1^2	$2\cdot 1k$	k^2+1^2
k^2-2^2	$2\cdot 2k$	k^2+2^2
k^2-3^2	$2\cdot 3k$	k^2+3^2
k^2-4^2	$2\cdot 4k$	k^2+4^2
k^2-r^2	$2\cdot rk$	k^2+r^2

From an examination of this table it becomes apparent that the sides follow some definite law which is as follows:

If any two values for k and r be selected, a right triangle may always be formed by substituting in the following k^2-r^2, one side, $2kr$ other side and k^2+r^2 hypotenuse, for $(k^2-r^2)^2+(2kr)^2 = (k^2+r^2)^2$ provided k is larger than r.

For example let $k = 7$ and $r = 4$. Substituting in the values in the formulae for sides we have $49-16$ or 33, $2\times 7\times 4$ or 56 and $49+16$ or 65 and squaring out we have $33^2+56^2 = 65^2$

or $\quad 1089+3136 = 4225.$

Since k^2-r^2 is one side and $2kr$ is the other, the area of the triangle may be found by multiplying them together and dividing by two, obtaining for the area of the triangle $kr(k^2-r^2)$.

The above formulae provide for an infinite number of right triangles which the instructor can make up almost instantly and yet have the values for his sides known to himself before the problem is given.

One more interesting fact is noticeable. If $k = 2r$ the sides are in the ratio 3, 4, and 5. This is also the case where $k = 3r$. From there on the ratios vary for if $k = mr$, substituting in the formulae for the sides, we have $(m^2r^2 - r^2)$, $(2mr^2)$ and $(m^2r^2 + r^2)$. Taking out a common factor of r^2, we have for the three sides, $r^2(m^2 - 1)$, $r^2(2m)$, and $r^2(m^2 + 1)$ and the ratios are therefore $m^2 - 1$, $2m$ and $m^2 + 1$.

When m is 1 ratios are	0, 2, 2 (impossible)
2	3, 4, 5
3	8, 6, 10 or 4, 3, 5
4	16, 8, 17
5	24, 10, 26
6	35, 12, 37

and so on, the triangles becoming closer and closer to isosceles right triangles but having the two longer sides always differing by 2.

[Note by Math. Ed.: For a full discussion of this problem see the following numbers of this journal: November, 1910; April, 1911; April, 1913.]

GOVERNMENT REPORT ON THE SUNSET-MIDWAY OIL DISTRICT OF CALIFORNIA.

The immense growth in the world demand for petroleum and its products is taxing to the uttermost the capacity of the known oil fields and stimulating the search for new ones. The study of the producing fields is being carried on more and more intensively, not only to learn what contributions may be expected from such new fields as may be found but to maintain the declining production of the old fields and to squeeze the last profitable drop from them.

The work of finding petroleum and of producing it is becoming less and less "a gamble" and more and more an exact science. Geologic work is not only essential to the discovery of new fields but can be profitably continued to direct exploration and production in old fields. For the lack of geologic work different producing beds reached by different wells have been confused during the early life of a field, and the possible sources of water trouble have been overlooked.

Men who might have been millionaires have been made practically penniless because they abandoned a well too soon through ignorance or misconception of the true number and position of the oil-bearing beds, and companies well on the road to success have suffered serious reverses by useless expenditure incurred in drilling wells where there was practically no chance of striking oil, and by deepening holes that had already passed through all the possible oil-bearing beds. Entire fields have been abandoned, or almost abandoned, only to be reopened when applied science or random drilling revealed the existence of oil sands below the old "pay." Other fields have been lost through water trouble due to carelessness, lack of experience, or ignorance of the true source of the trouble and how to prevent it.—[*U. S. Geological Survey.*

DR. IRVING LANGMUIR RECEIVES NICHOLS MEDAL.

The William H. Nichols Gold Medal presented annually by the New York Section for the best original paper printed in the publications of the American Chemical Society was conferred for 1919 last evening (Friday, March 5) upon Dr. Irving Langmuir, a noted physical chemist connected with the General Electric Company, of Schenectady, New York, for his paper entitled, "The Arrangement of Electrons in Atoms and Molecules." The subject is of far reaching importance on account of the belief in scientific circles that the world is on the verge of discovering methods for utilizing the force pent up in the atom along the lines suggested by Sir Oliver Lodge, Sir Ernest Rutherford and other distinguished physicists. The conferring of the award took place in Rumford Hall, 50 East 41st Street, at the monthly meeting of the New York Section, the speech of presentation being made by Dr. Nichols, former president of the Society and donor of the medal. After the acceptance Dr. Langmuir delivered an address entitled, "Octek Theory of Valence."

The jury in reporting its decision stated that, in view of the award of the same honor to Dr. Langmuir in 1915, it took special pleasure in thus again giving recognition to his continued valuable services to chemical science.

Dr. Langmuir was born in Brooklyn in 1881. He was graduated as a metallurgical engineer from Columbia University in 1903 and three years later received his doctor's degree from the University of Gottingen. He was for several years an instructor in chemistry in the Stevens Institute of Technology, Hoboken. He has done much research work in gas reactions, water vapors, iron pipe corrosions, and more recently in the application of tungsten to the manufacture of electric lamps.

COAL MINING IN ALASKA.

The Matanuska coal field, in Alaska, is just now of special interest because it is tributary to the Government railroad and because of the coal-mining developments carried on by the Department of the Interior through the Alaskan Engineering Commission. The United States Geological Survey, of the Department of the Interior, has therefore issued, in its Bulletin 712-E, a paper by Theodore Chapin entitled "Mining Developments in the Matanuska Coal Field." Mr. Chapin is in charge of the branch office of the Geological Survey at Anchorage, Alaska, and is actively cooperating with the Alaskan Engineering Commission in the development of these mines. His report is part of the Geological Survey's regular annual account of mining in Alaska and is based on several weeks' field work done at different times in the summer and fall of 1918 and on an intimate knowledge of the mining and prospecting that were done in that year. The report includes a brief description of the general geology of the coal field, a statement of the results of tests and analyses of the coal, a detailed account of the mining and prospecting done in 1918, and descriptions of the geologic features at the mines and maps of areas near them, as well as many sections showing the geologic relations of the coal beds. The report is supplemental to the more general accounts of the geology and coal of the Matanuska Valley that have already been published by the Geological Survey.

The same bulletin contains also a short paper by Mr. Chapin entitled "Lode Developments in the Willow Creek District," which is the customary brief annual statement of the developments of the year of the gold lode mines of that district.

Bulletin 712-E can be obtained free of charge from the Director of the Geological Survey at Washington, D. C.

DIRECTORY OF SCIENCE AND MATHEMATICS SOCIETIES.

Under this heading are published in the March, June, and October issues of this Journal the names and officers of such societies as furnish us this information. We ask members to keep us informed as to any change in the officiary of their society. This is extremely valuable information to all progressive teachers. Is your Society listed here? Names are dropped when they become one year old.

AMERICAN GEOGRAPHICAL SOCIETY OF NEW YORK.

President, John Greenough; *Vice-Presidents*, Paul Tuckerman and James B. Ford; *Foreign Corresponding Secretary*, William Libbey; *Domestic Corresponding Secretary*, Archibald D. Russell; *Recording Secretary*, Hamilton Fish Kean; *Treasurer*, Henry Parish; *Director and Editor*, Isaiah Bowman, Ph.D.—1220.

AMERICAN PHYSICAL SOCIETY.

President, J. S. Ames, Johns Hopkins University, Baltimore, Md.; *Vice-President*, Theodore Lyman, Harvard University, Cambridge, Mass.; *Secretary*, D. C. Miller, Case School of Applied Science, Cleveland, Ohio; *Treasurer*, G. B. Pegram, Columbia University, New York City; *Editor*, F. Bedell, Cornell University, Ithaca, N. Y.; *Local Secretary for the Pacific Coast*, E. P. Lewis, University of California, Berkeley, Cal. *The Council of the Society consists of the President, Vice-President, Secretary, Treasurer, Managing Editor, all living Past Presidents and eight elected members as follows: Past Presidents:* A. A. Michelson, A. G. Webster, Carl Barus, E. L. Nichols, Henry Crew, W. F. Magie, Ernest Merritt, R. A. Millikan, H. A. Bumstead; *Elected Members:* Max Mason, G. O. Squier, H. A. Wilson, A. L. Day, G. F. Hull, G. K. Burgess, J. C. McLennan, F. B. Jewett; *Editor*, Frederick Bedell, Ithaca, N. Y.—220.

AMERICAN PHYSIOLOGICAL SOCIETY.

President, Prof. Frederick S. Lee, Columbia University, New York; *Secretary*, Prof. Charles W. Greene, University of Missouri, Columbia, Mo.; Next annual meeting place, Johns Hopkins Medical School—419.

AMERICAN SOCIETY OF NATURALISTS.

President, Dr. Jacques Loeb, Rockefeller Institute for Medical Research, New York City; *Vice-President*, Prof. Bradley M. Davis, University of Michigan, Ann Arbor; *Secretary*, Prof. A. Franklin Shull, University of Michigan, Ann Arbor; *Treasurer*, Dr. J. Arthur Harris, Carnegie Institution for Experimental Evolution, Cold Spring Harbor, Long Island, N. Y.—220.

AMERICAN SOCIETY OF ZOOLOGISTS.

President, Gilman A. Drew, Marine Biological Laboratory, Woods Hole, Mass.; *Vice-President*, Caswell Grave, Washington University, St. Louis, Mo.; *Secretary-Treasurer*, W. C. Allee, Lake Forest College, Lake Forest, Ill.—220.

ASSOCIATION OF AMERICAN GEOGRAPHERS.

President, Charles R. Dryer; *First Vice-President*, Dr. Herbert E. Gregory; *Second Vice-President*, Dr. Isaiah Bowman; *Secretary*, Dr. Oliver L. Fassig; *Treasurer*, Francois E. Matthes.—1219.

AMERICAN ASSOCIATION OF ECONOMIC ENTOMOLOGISTS.

President, Wilmon Newell, Gainesville, Fla.; *First Vice-President*, H. A. Gossard, Wooster, Ohio; *Second Vice-President*, E. M. Ehrhorn, Honolulu, H. T.; *Third Vice-President*, J. G. Sanders, Harrisburg, Pa.; *Fourth Vice-President*, F. B. Paddock, Ames, Iowa; *Secretary*, S. F. Burgess, Melrose Highlands, Mass.; *Editor*, E. Porter Felt, Nassau, Rens. Co., N. Y.—220.

ASSOCIATION OF TEACHERS OF MATHEMATICS IN THE MIDDLE STATES AND MARYLAND.

President, W. E. Breckenridge, Stuyvesant High School, New York City; *Vice-President*, Louisa M. Webster, Hunter College, New York City; *Secretary*, C. B. Walsh, Friends' Central School, Philadalphia, Pa.; *Treasurer*, Clarence P. Scobori, Polytechnic Prep. School, Brooklyn, N. Y.; *The Council*, William Betz, Rochester, N. Y.—220.

CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS.

(*Annual Meeting*, November 25, 26, 27, *at Englewood High School, Chicago.*)

President, J. A. Foberg, Crane Technical High School, Chicago; *Vice-President*, Frederick R. Gorton, State Normal College, Ypsilanti, Mich; *Treasurer*, Lewis L. Hall, Morgans Park High School, Chicago; *Corre-*

sponding Secretary, Ally L. Marlatt, University of Wisconsin, Madison; *Assistant Treasurer*, E. S. Tillman, High School, Hammond, Indiana; the *Recording Secretary*, Harry O. Gillet, the University of Chicago, was elected last year for a two-year term. *Section Officers for 1920:*

Biology—Chairman, Grace J. Baird, Bowen High School, Chicago; *V.-Chairman*, C. P. Shideler, Joliet Twp. High School, Joliet, Ill.; *Secretary*, Elizabeth Foss, North High School, Minneapolis, Minn.; *Chemistry—Chairman*, B. J. Rivett, Northwestern High School, Detroit, Mich.; *V.-Chairman*, George Sype, Austin High School, Chicago; *Secretary*, K. J. Stouffer, Wayland Academy, Beaver Dam, Wis.; *General Science—Chairman*, G. A. Bowden, University School, Cincinnati, Ohio; *V.-Chairman*, George Mounce, LaSalle-Peru Twp. High School, LaSalle, Ill.; *Secretary*, Ruth C. Russell, Lake View School High, Chicago; *Geography—Chairman*, Wellington D. Jones, The University of Chicago, Chicago; *V.-Chairman*, R. R. Robinson, Joliet Twp. High School, Joliet, Ill.; *Secretary*, Anne B. Royeston, Hyde Park High School, Chicago; *Home Economics—Chairman*, Harriet Glendon, Lewis Institute, Chicago; *V.-Chairman*, Treva E. Kauffman, Ohio State University, Columbus, Ohio; *Secretary*, Maude M. Firth, Supervisor Home Econ., Davenport, Ia.; *Mathematics—Chairman*, M. J. Newell, High School, Evanston, Ill.; *V.-Chairman*, W. E. Beck, High School, Iowa City, Iowa; *Secretary*, Elsie G. Parker, Oak Park High School, Oak Park, Ill.; *Physics—Chairman*, H. Clyde Krenerick, North Div. High School, Milwaukee, Wis.; *V.-Chairman*, C. F. Phipps, State Normal School, DeKalb, Ill.; *Secretary*, Glenn W. Warner, Englewood High School, Chicago.—1119.

BOTANICAL SOCIETY OF AMERICA.

President, N. L. Britton, New York Botanical Garden; *Vice-President*, C. E. Allen, University of Wisconsin; *Secretary*, J. R. Schramm, Cornell University, Ithaca, N. Y.; *Treasurer*, Edmund W. Sinnott, Connecticut Agricultural College, Storrs, Conn.—320.

CHEMISTRY TEACHERS' CLUB OF NEW YORK CITY.

President, Alton I. Lockhart, Horace Mann School, New York City; *Vice-President*, J. Ellis Stannard, Boy's High School, Brooklyn, N. Y.; *Secretary*, Floyd L. Darrow, Polytechnic Preparatory Country Day School, Brooklyn, N. Y.; *Treasurer*, J. Hood Branson, Evander Childs High School, Bronx, N. Y.

ECOLOGICAL SOCIETY OF AMERICA.

President, Barrington Moore, American Museum of Natural History, New York City; *Secretary*, Dr. A. O. Weese, University of New Mexico, Albuquerque, N. M.—220

GENERAL SCIENCE CLUB OF NEW ENGLAND.

President, J. Richard Lunt, Boston English High School; *Vice-President*, W. H. D. Meier, Framingham Normal School; *Secretary*, Edith L. Smith, Boston Normal School; *Treasurer*, C. H. Stone, Boston English High School.—320.

ILLINOIS ACADEMY OF SCIENCE.

President, Henry B. Ward, University of Illinois; *Vice-President*, Geo. D. Fuller. University of Chicago; *Secretary*, J. L. Pricer, State Normal University, Normal; *Treasurer*, W. G. Waterman, Northwestern University.—319.

IOWA ACADEMY OF SCIENCE.

President, S. W. Beyer, Iowa State College, Ames; *First Vice-President*, T. C. Stephens, Morningside College, Sioux City; *Second Vice-President*, R. Monroe McKenzie, Parsons College, Fairfield; *Secretary*, James H. Lees, Iowa Geological Survey, Des Moines; *Treasurer*, A. O. Thomas, State University, Iowa City.—318.

INTERNATIONAL METALLURGICAL AND CHEMICAL SOCIETY.

President, Leo A. Cawes; *Vice-President*, Carl Valdemar; *Secretary* and *Editor*, Roy Franklin Heath, c-o Billings Polytechnic Institute, Billings, Montana.—320.

IOWA ASSOCIATION OF MATHEMATICS TEACHERS.

President, W. J. Rusk, Grinnell College, Grinnell, Iowa; *Vice-President*,

O. F. Nixon, Principal of High School, Fairfield, Iowa; *Secretary-Treasurer*, Ira S. Condit, Iowa State Teachers College, Cedar Falls, Iowa.—120.

IOWA SECTION OF THE MATHEMATICAL ASSOCIATION OF AMERICA.

President, Ira S. Condit, Iowa State Teachers College, Cedar Falls, Iowa; *Secretary*, Le Roy Coffin, Coe College, Cedar Rapids, Iowa.—120.

KANSAS ASSOCIATION OF MATHEMATICS TEACHERS.

President, J. A. G. Shirk, Pittsburg, Kansas, one year; *Vice-President*, Emma Hyde, Emporia, Kansas, one year; *Secretary-Treasurer*, Edna E. Austin, No. 6 The Devon, Topeka, Kansas, three years.—1119.

MATHEMATICAL ASSOCIATION OF AMERICA.

President, David Eugene Smith, Teachers College, Columbia University; *Vice-Presidents*, Helen A. Merrill, Wellesley College, Wellesley, Mass., and E. J. Wilczynski, University of Chicago, Chicago, Ill.; *Secretary-Treasurer*, W. D. Cairns, Oberlin College, 27 King Street, Oberlin, Ohio.—120.

NATIONAL EDUCATION ASSOCIATION.

President, Josephine Corliss Preston, Olympia, Wash.; *Secretary*, J. W. Crabtree, Washington, D. C.; *Treasurer*, A. J. Matthews, Tempe, Ariz.; *Department of Secondary Education: President*, W. W. McIntire, Principal Norwood High School, Norwood, Ohio; *Vice-President*, Charles H. Perrine, Principal Wendell Phillips High School, Chicago, Ill.; *Secretary*, Anna Willson, Principal High School, Crawfordsville, Ind.; *Department of Higher Education: President*, Guy Stanton Ford, University of Minnesota, Minneapolis, Minn.; *Secretary*, J. J. Pettijohn, Director Educational Extension, Bureau of Education, Washington, D. C.; *Department of Normal Schools: President*, John R. Kirk, President State Normal School, Kirksville, Mo.; *Secretary*, Anna M. Tibbetts, Fargo College, Fargo, N. D.; *Vice-President*, Joseph Rosier, President Fairmont State Normal School, Fairmont, W. Va.; *Department of Science Instruction: President*, George R. Twiss, Ohio State University, Columbus, Ohio; *Vice-President*, James H. Smith, Assistant Principal, Austin High School, Chicago, Ill.—719.

MISSOURI SOCIETY OF TEACHERS OF MATHEMATICS AND SCIENCE.

Department of Mathematics and Science: President, O. M. Stewart, Columbia, Mo.; *Secretary*, B. F. Finkel, Drury College, Springfield, Mo.; *Treasurer*, A. J. Schwartz, St. Louis, Mo.; *Mathematics Division: Vice-President*, W. A. Luby, Kansas City; *Secretary*, Eula Weeks, St. Louis, Mo.—419.

NEW ENGLAND ASSOCIATION OF CHEMISTRY TEACHERS.

President, William W. Obear, High School, Somerville, Mass.; *Curator*, Lyman C. Newell, Boston University, Boston, Mass.; *Secretary*, S. Walter Hoyt, Mechanic Arts High School, Boston, Mass.; *Treasurer*, Alfred M. Butler, High School of Practical Arts, Boston, Mass.—120.

NEW ENGLAND ASSOCIATION OF TEACHERS OF MATHEMATICS.

President, William R. Ransom, 29 Sawyer Ave., Tufts College, Mass.; *Vice-President*, Edmund D. Searles, High School, New Bedford, Mass.; *Secretary*, Harry D. Gaylord, 448 Audubon Road, Boston, Mass.; *Treasurer*, Harold B. Garland, High School of Commerce, Boston, Mass.—1218.

NEW JERSEY SCIENCE TEACHERS' ASSOCIATION.

President, Merton C. Leonard, Dickinson H. S., Jersey City; *Vice-President*, Walter J. Dumm, Barringer H. S., Newark; *Secretary-Treasurer*, Carl J. Hunkins, South Side H. S., Newark; *Recording-Secretary*, Agnes V. Luther, Newark Normal School, Newark; *Chairman Biology Section*, F. H. Hodgson, High School, Montclair; *Secretary Biology Section*, Miss L. H. Seeley, Dickinson H. S., Jersey City; *Chairman Chemistry Section*, C. E. Dull, South Side H. S., Newark; *Secretary Chemistry Section*, Miss Pauline McDowell, Battin H. S., Elizabeth; *Chairman Physics Section*, W. H. Platzer, High School, Kearny; *Secretary Physics Section*, Carl O. Voegelin, Central C. & M. T. H. S., Newark; *Chairman Elementary Science Section*, Miss Edith L. Spencer, South Side H. S., Newark; *Secretary Elementary Science Section*, Miss Julia G. Pierce, High School, Paterson; *Members at large of the Executive Committee*, Dr. M. T. Cook, Rutgers College, New Brunswick, and Miss Edith S. Merritt, Dickinson H. S., Jersey City.—120.